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Module 2

Applied Mathematics 30

MATRICES

Learning Technologies Branch

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Module
2

Applied Mathematics 30

MATRICES

Learning
Technologies
Branch

Alberta
LEARNING

Applied Mathematics 30
Module 2: Matrices
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-2183-1

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

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Welcome

Applied Mathematics 30

Welcome to Module 2.
We hope you'll enjoy your study of Matrices.

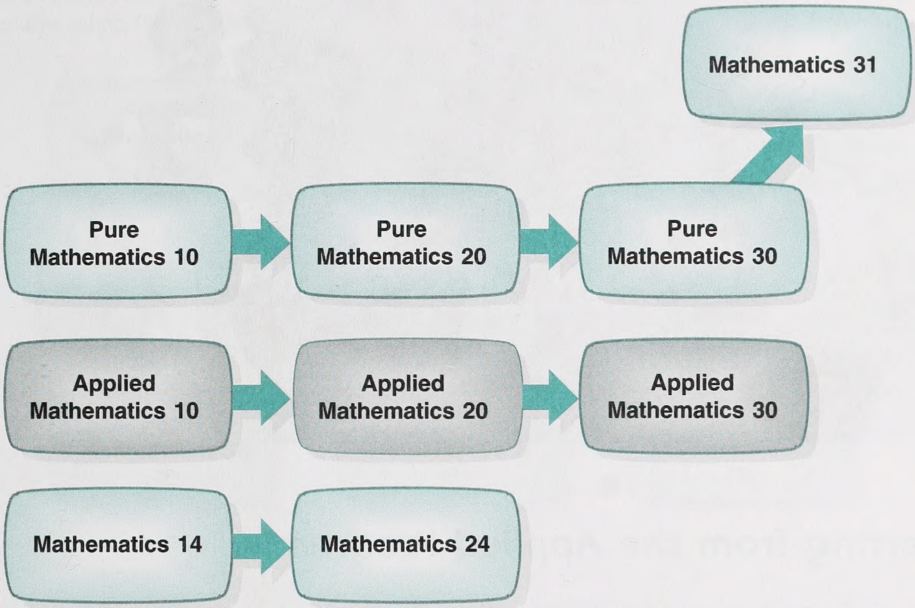
- Module 1: Probability
- **Module 2: Matrices**
- Module 3: Statistics
- Module 4: Personal Finance
- Module 5: Sinusoidal Data
- Module 6: Patterns
- Module 7: Vectors

Applied Mathematics 30 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

Contents

Introduction to Applied Mathematics 30

Applied Mathematics 30 is the third course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.



Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different career plans. For example, Pure Mathematics 30 is a prerequisite for admission to many university programs. Many colleges and technical institutes, however, will admit students who have successfully completed Applied Mathematics 30.

You may find it helpful to read any of the documents under the heading "New Senior High School Mathematics Update/Post-Secondary Studies Update" at the following Internet site:

http://www.learning.gov.ab.ca/k_12/curriculum/bySubject/math

Before enrolling in Applied Mathematics 30, it is recommended that you talk with a school counsellor about your career plans.

Transferring from the Applied Program

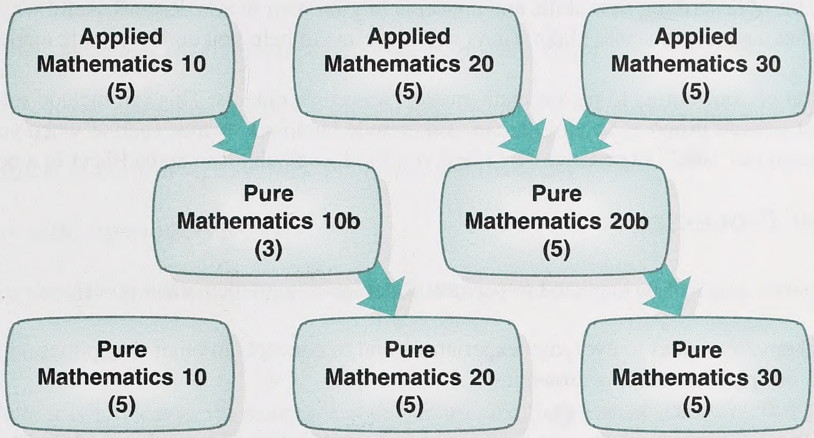
You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are independent.

The following table shows some common and independent topics.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectations • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics.

If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may have to take a three-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20 or Applied Mathematics 30, you may have to take a five-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.

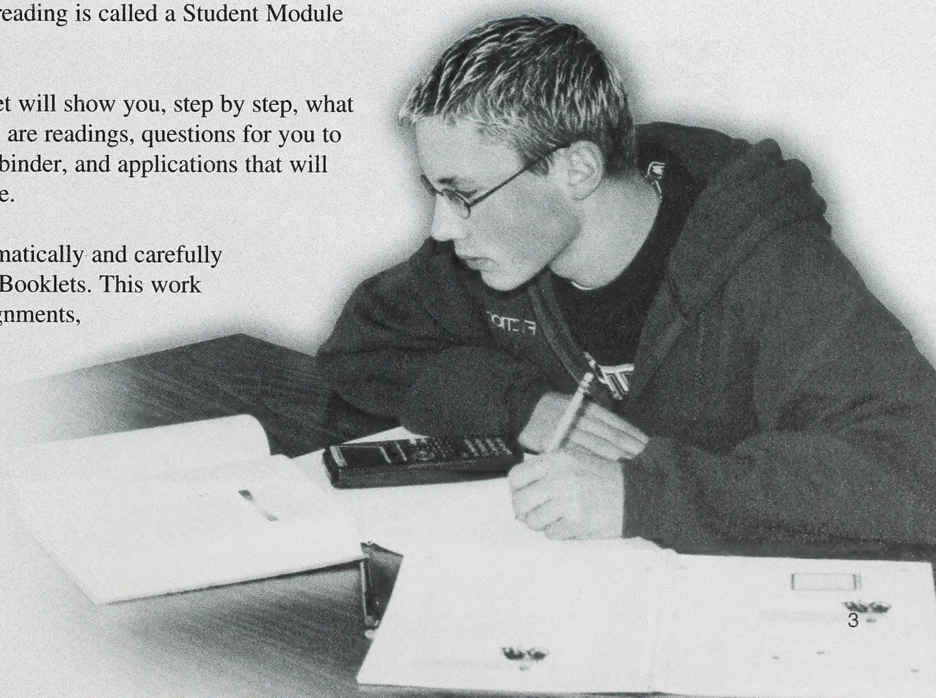


Strategies for Completing Applied Mathematics 30

For each module in Applied Mathematics 30, there is a Student Module Booklet and accompanying Assignment Booklets. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step by step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the assignments, projects, and final test.



Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also store your marked assignments here.
- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.
- Keep a section of your binder to record new skills and concepts, as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words. Record useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.
- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an "aha!" experience), or when you used a calculator or spreadsheet in a new way.

Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 30 to be your study partner. You will find that having a friend to discuss mathematical ideas with will make your studying more enjoyable.

Resources You Will Need

In addition to the course materials for Applied Mathematics 30, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 12 Source Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (2002)

- the *Addison-Wesley Applied Mathematics 12 Project Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (2002)

- a binder, lined loose-leaf paper, graph paper, dividers, pencils, eraser

- metric and imperial measuring devices, such as a ruler, yardstick, metre-stick, and tape measure

- a mathematical instrument set (compass, protractor, and triangles)

- a computer with a spreadsheet program

  Note: Two popular spreadsheet programs are *ClarisWorks*™ and Microsoft® *Excel*.

- a graphing calculator

  Note: Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

Texas Instruments	Sharp	Casio	Hewlett-Packard
TI-83 TI-83 Plus TI-86 TI-89 TI-92* TI-92 Plus	EL-9600C EL-9600* EL-9200* EL-9300*	Algebra FX 2.0 CFX-9850 GA-Plus* CFX-9850 G* CFX-9800 G* FX-9700 series*	HP 39g†

*no longer commercially available
†The HP 39g calculator will remain on the approved list for the 2001–2003 school years and will then be deleted from the approved list.

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you obtain the video program *The TI-83 Graphing Calculator Video Tutor*.

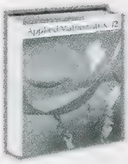
Many of the resources you will need may be purchased locally or from the Learning Resources Centre (LRC). Following is the LRC website:

http://www.lrc.learning.gov.ab.ca

You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

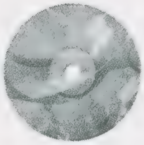
Visual Cues

You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the following explanations to discover what the various icons prompt you to do.

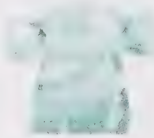


Refer to the textbook or the Project Book.

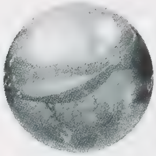
Work with a computer.



Refer to the Applied Mathematics 30 CD.



Contact your teacher for additional information.



Explore the Internet.

Complete specified questions in the Assignment Booklet.

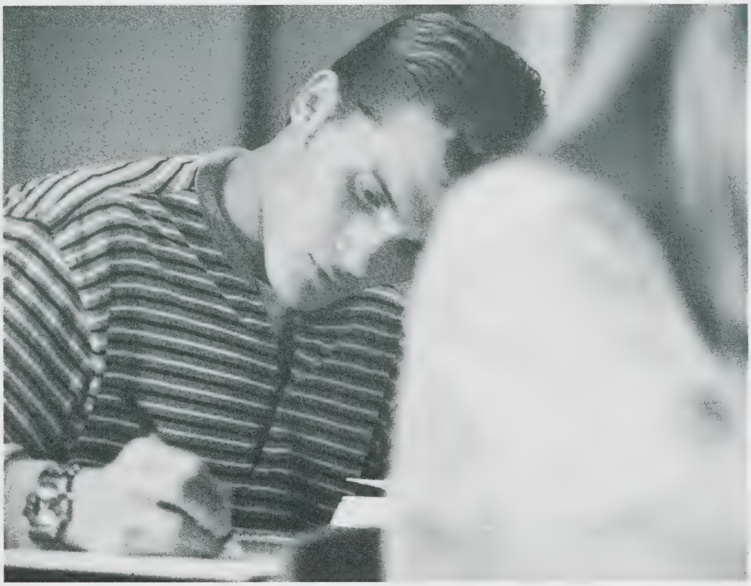
Remember: Any Internet website address given in this module is subject to change.

Where Can I Obtain Diploma Examination Information?

Alberta students will write a diploma examination at the end of the course. Alberta Learning provides several documents to help students prepare for this examination. These documents are found under the heading "Diploma Examinations" at the following Alberta Learning website:

http://www.learning.gov.ab.ca/k_12/testing

Information like course expectations, the makeup of the diploma examination, keyed copies of previous examinations, preparation guides, and calculator policies are available to students at this site.



Each year, in February and September, Alberta Learning provides teachers with information on a **student project**, which teachers **may** use as part of your overall assessment. Information to students will also be posted on the Alberta Learning website. Check with your teacher to determine what you will be expected to do. Be aware that one of the diploma examination's written-response questions will deal with elements of this project and is worth 10% of your diploma examination mark.

You should take advantage of the many sources of information about Applied Mathematics 30. Your success depends on your understanding of course expectations and evaluation procedures. Work closely with your teacher and do not hesitate to ask questions.

Remember, take the initiative to find out all you can about Applied Mathematics 30.

MODULE OVERVIEW

Have you ever wondered, if you have mailed a letter or parcel to a friend or relative in another part of the country, how long it would take for the letter or parcel to reach its destination and how many people would handle that parcel on route? If you live in a smaller community, the letter or parcel is first transported, by truck, from the post office in your community to a large distribution centre. Once there, the letter or parcel is sorted and shipped to the large distribution centre in the part of the country in which the letter or parcel is addressed. Once it arrives at the second distribution centre, the letter or parcel is sorted, once again, and delivered to the address written on the letter or parcel. Figuring out the route your letter or parcel takes is an example of a pathway problem that can be analysed by arrays, called matrices.

In this module, you will investigate matrices and how they are used to present and analyse information. In particular, you will model problems using matrices and, as part of your investigation, you will use technology to perform operations on these matrices. The problems you will explore will include networks and probabilities.

If you are required to do so, you will use matrices in your module project to analyse long-distance telephone plans. The skills you acquire throughout this module will assist you in this exploration.

Assessment

Accompanying this Student Module Booklet are two Assignment Booklets. Your grading in this module will be based upon the assignments you submit for assessment. The mark distribution is as follows:

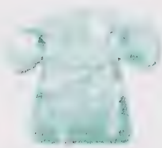
Assignment Booklet 2A	
Activities 1 and 2 Assignment	**70 marks**
Assignment Booklet 2B	
Activities 3 and 4 Assignment	**70 marks**
Module Review	**20 marks**
Module Project	**40 marks**
TOTAL	**200 marks**

Remember that Activities 1 to 4 in this Student Module Booklet will prepare you for completing the module project and the module assignment. You should work through these activities carefully and compare your answers with the suggested answers provided in the Appendix.

The Module Review provides a review of the module and an enrichment activity. You may choose to do some or all of the questions in the Module Review. Again, you should compare your answers with the suggested answers provided in the Appendix.

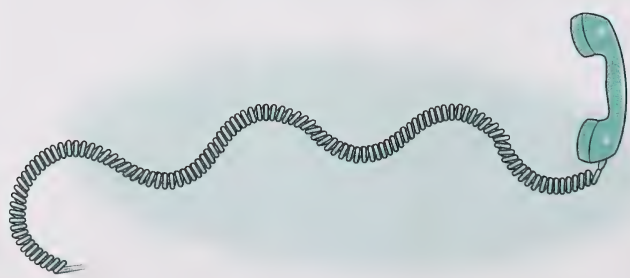
Module Project
Creating a Long-Distance Telephone Plan

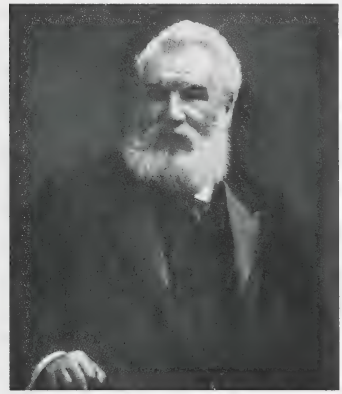
Beginning the Project



Your teacher may not require you to complete all the projects provided in this Applied Mathematics 30 course. Contact your teacher and check whether you need to complete the module project, Creating a Long-Distance Telephone Plan, as part of your assessment.

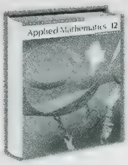
Today, most people take electronic communication for granted. Using long-distance telephone services, e-mail, and faxes is an important part of daily personal and business routines. It is hard to believe that these devices are comparatively recent developments.



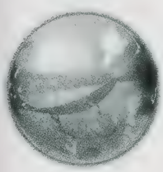

Alexander Graham Bell (1847–1922)

Alexander Graham Bell invented the telephone in 1876—he tested it in Brantford, Ontario. In Eastern Canada, it wasn't until the early 1880s that the Bell Telephone Company of Canada began developing the necessary infrastructure for personal telephone service. In Western Canada, in 1908 and 1909, the governments of Alberta, Saskatchewan, and Manitoba purchased Bell's operations and operated telephones as provincial utilities. In the 1990s, long-distance service was opened to competition. Now, long-distance providers compete for clients, trying to entice people with incentives and attractive rate structures.

The Module 2 project, Creating a Long-Distance Telephone Plan, involves examining the various plans offered by long-distance providers in your area. You will use matrices to analyse these plans and determine which plan best serves the long-distance needs of your family. You will summarize the results of your investigation, propose your own plan based on these results, and survey telephone users to predict the number of people in your area who may want to adopt your plan.



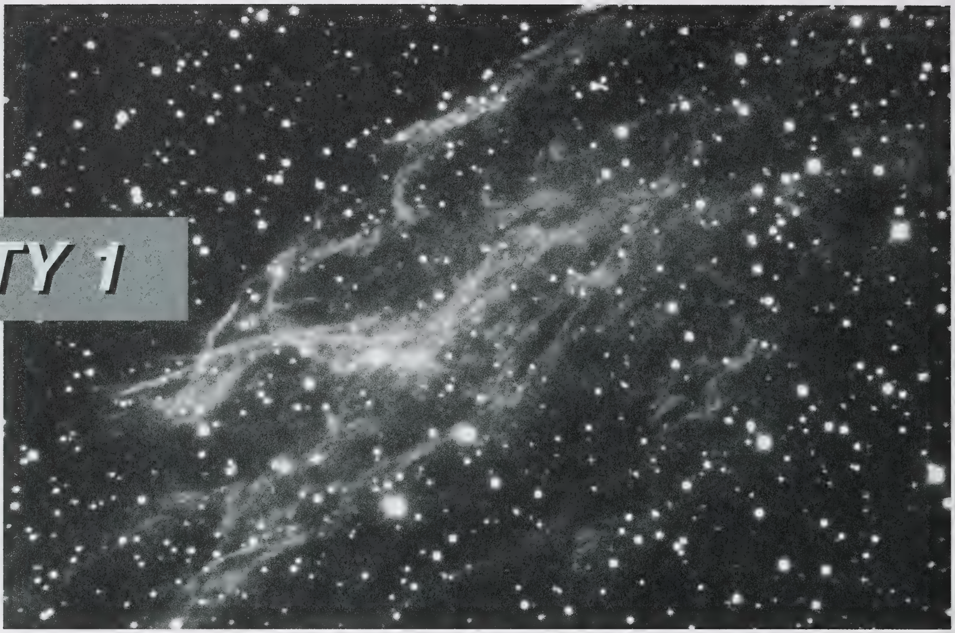
Turn to page 50 of the textbook and read "Creating a Long-Distance Telephone Plan." Answer the questions posed, and store your responses in the project section of your mathematics binder.



After you have recorded some of your initial ideas regarding the project, begin researching some of the things you may need to know about long-distance service plans. To start, you may find it useful to visit Addison-Wesley's Internet site, described on page 51 of the textbook. **Note:** The topics are listed under the heading "Chapter 2: Creating a Long-Distance Telephone Plan," not "Long Distance Plans." This website provides links to several sites you may find helpful in researching long-distance plans.

As you work through Activities 1 to 4, continue to research ideas for long-distance telephone plans by surfing the Internet, calling competing providers for the details of their long-distance packages, or reading newspaper advertisements or promotional brochures. Concepts presented in this module, such as matrix multiplication and using transition matrices, will be useful in completing this project.

You will be given more direction on how to complete this project later in this module. In the meantime, feel free to discuss your project with your study partner or a family member. Remember, the work on the project you submit must be your own.

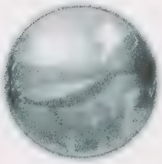


ACTIVITY 1

Modelling Problems with Matrices

The night sky is a treasure-trove of interesting and beautiful sights. People have been looking at the objects in the sky and trying to explain them since before the start of recorded history. Scientists have always been looking for simple, concise, and clear explanations of the phenomena that are around us. In the seventeenth century, Sir Isaac Newton proposed what has become known as the Law of Universal Gravitation as a means of explaining the motion of the planets and moons. In the early twentieth century, Albert Einstein's theory of relativity extended the theoretical physicist's understanding of how the universe worked. Dr. Einstein spent most of his remaining years trying to build a unified theory that explained both the large-scale events of the universe and the extremely small-scale events at a sub-molecular level. Now, at the turn of the twenty-first century, there seems to be a theory that will, indeed, explain all of these things. It goes by varying names, one of which is Matrix Theory.

In this module, you will be studying matrices in a more classic fashion. This is more in keeping with the methods used by Arthur Cayley, a mathematician who devised matrices to simplify the notation of solving systems of equations.

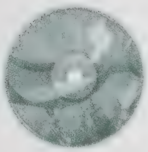


If you have access to the Internet, you can find out more about Arthur Cayley at the following website:

http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Cayley.html

Quite simply, a **matrix** is a rectangular array of **elements**; these elements are often numbers. For example, the following matrix, M, consists of a rectangular array of 12 numbers arranged in 3 rows and 4 columns.

$$M = \begin{bmatrix} 7 & -2 & 5 & 6 \\ 6 & 0 & -1 & 8 \\ 9 & 4 & 1 & 0 \end{bmatrix}$$



To discover where matrices are used in science and industry, view the segment titled *Matrices* on the Applied Mathematics 30 CD.

You will now become familiar with some of the terminology associated with matrices.

Turn to page 52 of the textbook and read the introductory paragraphs of Tutorial 2.1, "Matrix Operations." Pay particular attention to the following terms: **dimension**, **column matrix**, **row matrix**, and **square matrix**. Then work through "Example 1: Create a Matrix from a Table" on pages 52 and 53.

1. Answer exercises 1, 2, and 3 of "Discussing the Ideas" on page 55 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 51–52.

Now that you are familiar with the form and some of the terms associated with matrices, you will investigate how to add, subtract, and multiply matrices. You will begin with matrix addition and subtraction.

Suppose A and B are these 2×2 matrices:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad B = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$$

The addition of A and B is defined as follows:

$$\begin{aligned} A + B &= \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} w & x \\ y & z \end{bmatrix} \\ &= \begin{bmatrix} a+w & b+x \\ c+y & d+z \end{bmatrix} \end{aligned}$$

The subtraction of B from A is defined as follows:

$$A - B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} w & x \\ y & z \end{bmatrix}$$
$$= \begin{bmatrix} a-w & b-x \\ c-y & d-z \end{bmatrix}$$

You can use your graphing calculator to perform these operations. Study the following example.

Example

$$A = \begin{bmatrix} 1 & 4 \\ -3 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & -2 \\ 1 & -5 \end{bmatrix}$$

a. Determine $A - B$ using the definition for the subtraction of matrices.

b. Determine $A - B$ using your graphing calculator.

Solution

a. You can determine $A - B$ because they have the same dimension. They are both 2×2 matrices.

$$\therefore\ A - B = \begin{bmatrix} 1 & 4 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 6 & -2 \\ 1 & -5 \end{bmatrix}$$
$$= \begin{bmatrix} 1-6 & 4-(-2) \\ -3-1 & 5-(-5) \end{bmatrix}$$
$$= \begin{bmatrix} 1-6 & 4+2 \\ -3-1 & 5+5 \end{bmatrix}$$
$$= \begin{bmatrix} -5 & 6 \\ -4 & 10 \end{bmatrix}$$

b. Before starting, clear any matrices in your calculator's memory. Press 2nd [MEM] 2 (2:Delete…) 5 (5:Matrix…). Then press ENTER repeatedly until there are no matrices listed.

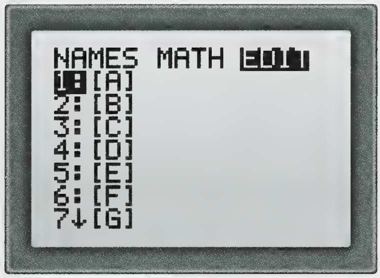
To use your graphing calculator, follow these steps:

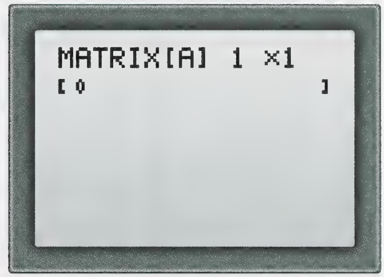
Step 1: Enter matrix *A*.

- Select matrix *A*.

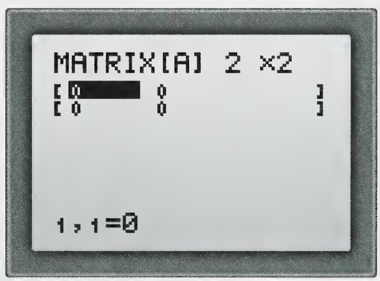
Select the EDIT menu.

MATRX ◀ 1 (1:[A])





- Change the expression "1×1" at the top of the screen to "2×2." This indicates that there are 2 rows and 2 columns in this matrix.



- Enter the elements of the matrix going across the first row and then across the second row.

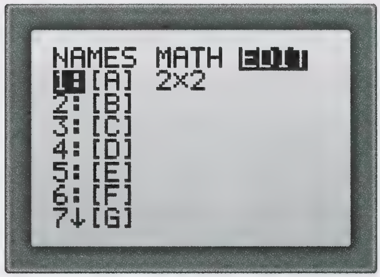
1 ENTER 4
ENTER (–) 3
ENTER 5 ENTER

Step 2: Enter matrix *B*.

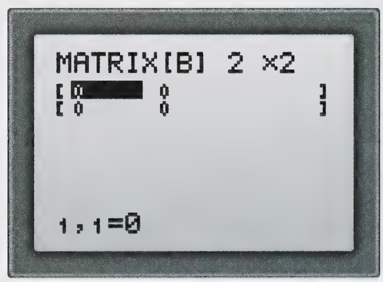
- Select matrix *B*.

Select the EDIT menu.

MATRX ◀ 2 (2:[B])



- Change the expression "1×1" at the top of the screen to "2×2 ."



- Enter the elements of the matrix.

6 ENTER (–) 2
ENTER 1 ENTER
(–) 5 ENTER

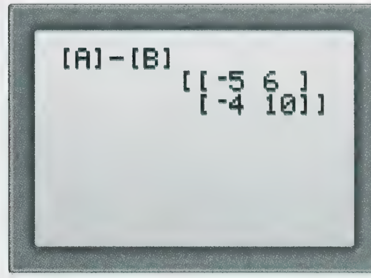
- Press 2nd [QUIT] to quit the entry of *B*.

Step 3: Determine $A-B$.

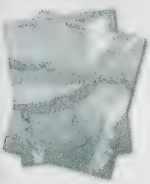
MATRX 1 (1:[A] 2×2)

− MATRX 2 (2:[B] 2×2)

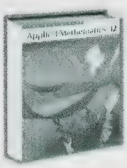
ENTER



$$\therefore\ A-B=\begin{bmatrix}-5 & 6\\ -4 & 10\end{bmatrix}$$

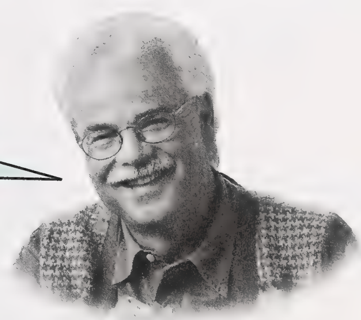


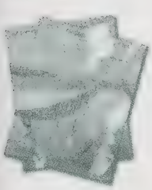
Turn to page 1 of Assignment Booklet 2A
and answer question 1.



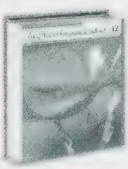
Turn to page 53 of the textbook and read the text following Example 1. Then work through “Example 2: Add Matrices” on pages 53 and 54 of the textbook.

In the previous examples, you discovered that you can only add or subtract matrices with the same dimension. **Remember**, only add or subtract the corresponding elements in the two matrices.



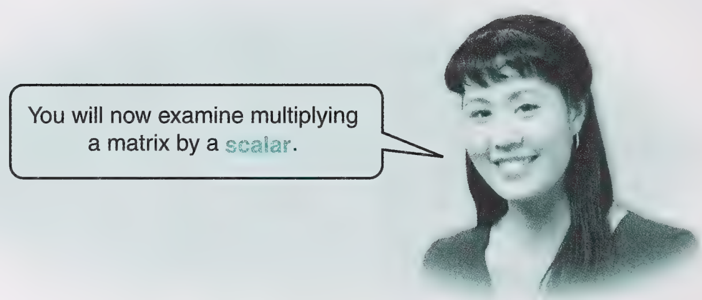


Turn to page 1 of Assignment Booklet 2A
and answer question 2.



2. Turn to pages 55 and 56 of the textbook and answer exercises 1.a., 1.b., and 3 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 52–53.

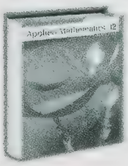


Turn to page 54 of the textbook and read the paragraph preceding Example 3. Then work through "Example 3: Use Technology to Multiply a Matrix by a Scalar" on pages 54 and 55.

In Example 3, you multiplied a matrix by a scalar, which involved multipling each entry in the matrix by that scalar. This can be summarized as follows:

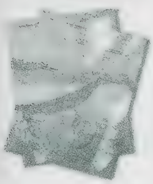
$$\text{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ then } kA = k\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$= \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

In the exercises that follow, you will encounter several additional applications of scalar multiplication.



3. Turn to pages 55 to 58 of the textbook and answer exercises 1.c., 1.d., 2, 5.a., 5.b., 6, and 7 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 53–57.



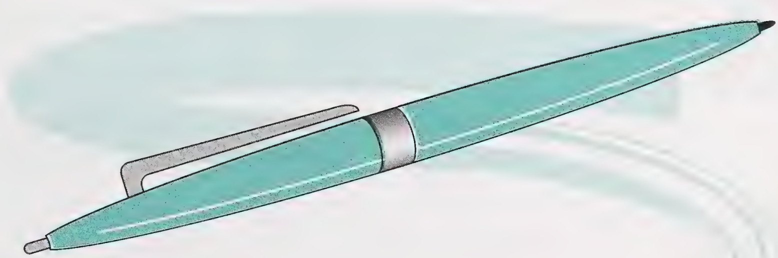
Turn to page 2 of Assignment Booklet 2A and answer questions 3, 4, and 5.

Looking Back

In this activity, you were introduced to matrices and their notation and terminology, matrix addition and subtraction, and the multiplication of a matrix by a scalar.

4. Turn to page 59 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 1, page 58.



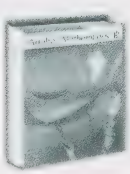
ACTIVITY 2

CORBIS/MAGMA

Matrix Multiplication

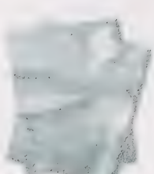
Many people enjoy playing video games. The images in many of these games mimic reality. Did you know that matrices play an important role in computer graphics? Programmers use matrices to transform images—to rotate, reposition, reflect, or distort them. Mathematicians developed the theory of transformations well before the invention of computers and computer graphics. If you continue your study of mathematics after high school, you will discover that successive transformations involve matrix multiplication. As a matter of fact, the application of matrices to transformations led directly to the definition of matrix multiplication that Cayley introduced in 1855.

In this activity, you will explore matrix multiplication using paper-and-pencil techniques and your graphing calculator. You will also investigate a variety of applications of matrix multiplication.



1. Turn to page 60 of the textbook and complete exercises 1 to 6 of "Investigation 1: Multiplying Matrices."

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 58–59.

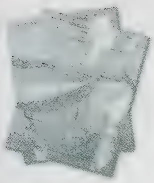


Turn to page 3 of Assignment Booklet 2A and answer question 6.

Turn to page 61 of the textbook and read the text preceding Investigation 2. Pay particular attention to how the elements of matrices A and B are combined to determine $A \times B$.

2. Complete exercises 1, 2, 3, and 5 of "Investigation 2: Dimensions of the Product Matrix" on pages 61 and 62 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 59–60.



Turn to page 3 of Assignment Booklet 2A and answer question 7.

In the preceding investigation, you discovered that in order to determine $A \times B$ for matrices A and B, the number of columns in A must be the same as the number of rows in B. In the next example, you will use your graphing calculator to check matrix multiplication.

Example

$$A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \\ 4 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

a. Find $A \times B$ using the definition of matrix multiplication.

b. Find $A \times B$ using your graphing calculator.

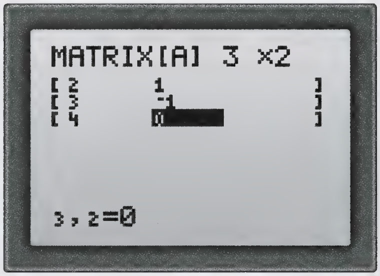
Solution

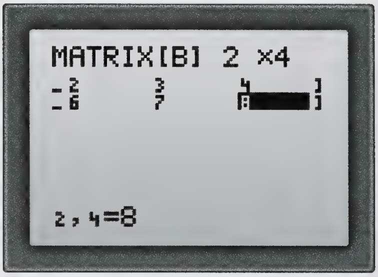
a. $$A \times B = \begin{bmatrix} (2\times1)+(1\times5) & (2\times2)+(1\times6) & (2\times3)+(1\times7) & (2\times4)+(1\times8) \\ (3\times1)+(-1\times5) & (3\times2)+(-1\times6) & (3\times3)+(-1\times7) & (3\times4)+(-1\times8) \\ (4\times1)+(0\times5) & (4\times2)+(0\times6) & (4\times3)+(0\times7) & (4\times4)+(0\times8) \end{bmatrix}$$

$$= \begin{bmatrix} 2+5 & 4+6 & 6+7 & 8+8 \\ 3+(-5) & 6+(-6) & 9+(-7) & 12+(-8) \\ 4+0 & 8+0 & 12+0 & 16+0 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 10 & 13 & 16 \\ -2 & 0 & 2 & 4 \\ 4 & 8 & 12 & 16 \end{bmatrix}$$

b. First, enter the elements of matrices A and B.

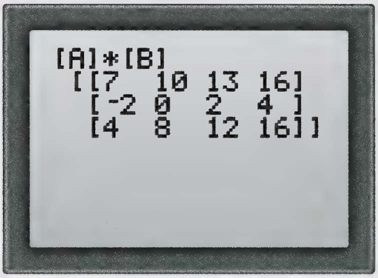


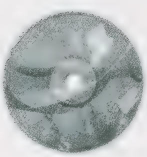


Now, determine $A \times B$. **Remember:** You must press [2nd] [QUIT] to exit the Matrix Edit feature.

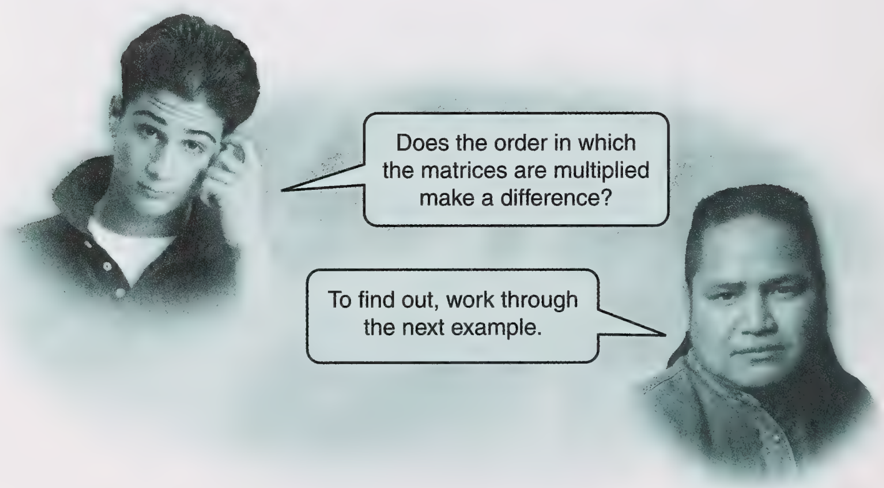
[MATRX] [1] (1:[A] 3×2) [×]

[MATRX] [2] (2:[B] 2×4) [ENTER]





For more information on matrices and their practical applications in the real world, view the segment *Iterations and Matrices* on the Applied Mathematics 30 CD. This segment discusses how to solve a problem using iterations or matrices. It also demonstrates how to use your graphing calculator to multiply matrices.



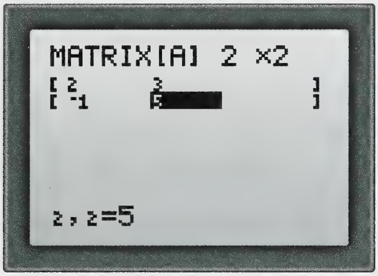
Example

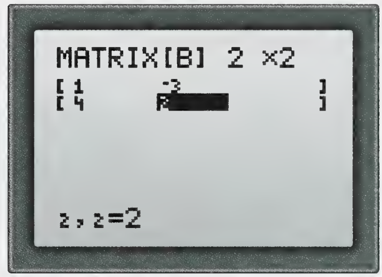
$$A = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$$

Using your graphing calculator, find the matrix products $C = AB$ and $D = BA$. Are products C and D the same?

Solution

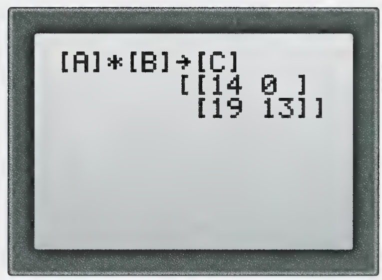
Define matrix A and matrix B.





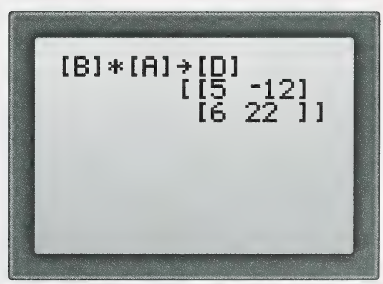
Find $C = AB$.

MATRX 1 (1:[A] 2×2) × MATRX 2 (2:[B] 2×2) STO➡ MATRX 3 (3:[C]) ENTER

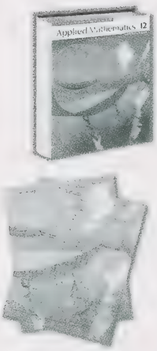


Find $D = BA$.

MATRX 2 (2:[B] 2×2) × MATRX 1 (1:[A] 2×2) STO➡ MATRX 4 (4:[D]) ENTER



Matrix C is not the same as matrix D.

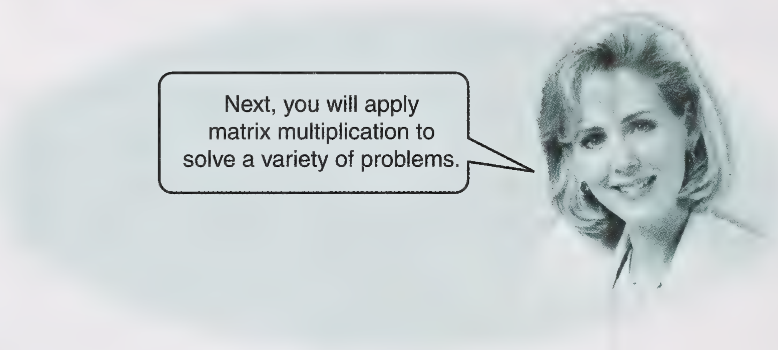


Turn to page 62 of the textbook and read the summary in the coloured box. Pay particular attention to the relationship among the dimensions of matrix A, matrix B, and the product matrix AB.

Turn to page 4 of Assignment Booklet 2A and answer question 8.

3. Answer exercises 1 and 2 of "Exercises: Checking Your Skills" on page 64 of the textbook. Use your graphing calculator for these exercises.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 61–63.

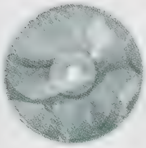


Turn to pages 62 and 63 of the textbook and work through "Example: Multiplying Matrices."

Turn to page 4 of Assignment Booklet 2A and answer question 9.

4. Answer exercises 3, 5, 7, and 9 of "Exercises: Checking Your Skills" on pages 64 to 67 of the textbook. Use your graphing calculator to do these exercises.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 63–71.

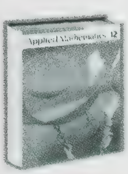


For an example of how to use estimation to assist in problem solving, view the segment *Simple Estimation Techniques* on the Applied Mathematics 30 CD.

You will now complete a selected project from the *Addison-Wesley Applied Mathematics 12 Project Book.* This project will involve writing matrices and matrix operations.

Project: Rustic Twig Furniture

Rustic furniture, popular in the nineteenth century in both Canada and the United States, is once again gaining popularity. Various items, such as chairs, tables, settees, swing seats, and planters, can be made from a variety of twigs or branches. In this project you will use matrices to track and analyse the quantity of materials required, the cost of labour and materials, the monthly orders, the selling price, and the profit for several different items. You will then analyse one particular item.



Turn to page 22 of the Project Book and read the information in the first two paragraphs of "Getting Started."

5. State two types of branches required to construct a chair.

6. How are the branches fastened together when twig furniture is produced?

Compare your responses with the suggested answers in the Appendix, Activity 2, page 72.

The lengths of the branches required and the number of nails needed for different types of furniture can be tracked using matrices. Turn to page 23 of the Project Book and read exercise 1 of "Getting Started."

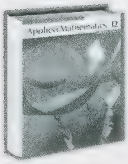
7. Copy and complete the tables in exercise 1 on page 23 of the Project Book. Use the charts in exercise 9 on pages 25 and 26 to complete Table A. Create matrix A from Table A.

8. Use the following as Table B.

Table B

	March	April	May	June
Chair	4	9	7	2
Settee	2	4	3	5
Side Table	3	6	7	3

   a. Create matrix B from Table B.



   b. Complete exercises 1.a. to 1.d. of "Getting Started" on page 23 of the Project Book.

9. Answer exercise 2 of "Getting Started" on page 23 of the Project Book.

10. Answer exercises 3.a. to 3.c. of "Getting Started" on page 24 of the Project Book; then complete the table in exercise 3 on page 24.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 72–73.

Now that you know the amount of time required to complete each process, you can determine the cost of labour. The labour rates are outlined in the following table.

Table E

	Collection	Construction	Finishing
Labour Rate	\$7/h	\$10/h	\$10/h

11. Create matrix E from Table E.

12. Answer exercises 4.a. to 4.c. of "Getting Started" on page 24 of the Project Book.

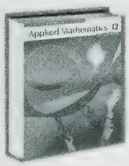
Compare your responses with the suggested answers in the Appendix, Activity 2, page 73.

Matrices can also be used to show the costs of purchased materials. Operations of matrices using your graphing calculator can then be used to determine the selling price and the profit of each item. The following table has been set up using approximately 100 nails/$\frac{1}{4}$-pound at a cost of \$1.19/pound and finishing oil/varnish at \$12.00/L.

Table G

	Chair	Settee	Side Table
Materials Cost	\$6.25	\$9.25	\$3.20

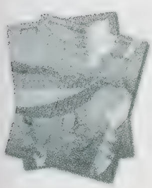
13. Create matrix G from Table G.



14. Matrix H shows that the total costs for material and labour can be obtained from matrices F and G. Explain how to create matrix H, and write it out.

15. Complete exercises 6, 7, and 8 of "Getting Started" on page 25 of the Project Book.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 73–74.



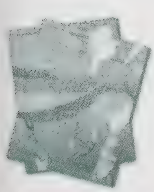
Turn to pages 5 and 6 of Assignment Booklet 2A and answer questions 10, 11, and 12.

Looking Back

In this activity, you explored matrix multiplication and its application to problem solving. You used pencil-and-paper techniques and your graphing calculator to determine these product matrices.

16. Turn to page 67 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 2, page 74.



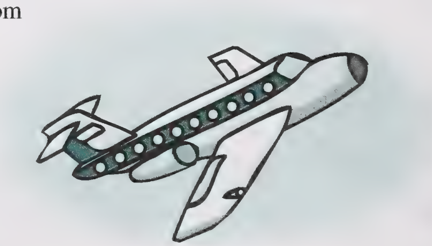
Turn to pages 6 to 10 of Assignment Booklet 2A and answer questions 13 to 18.

ACTIVITY 3

Solving Network Problems with Matrices

Have you considered a career as a travel agent? Travel agents, and the agencies they work for, are in the business of providing clients with advice, planning travel itineraries, booking transportation and accommodations, and providing any other preparatory help needed to assure a successful holiday or business trip. The main job of the travel agent, though, is to get the customers to their destinations with little delay and with minimal stopovers.

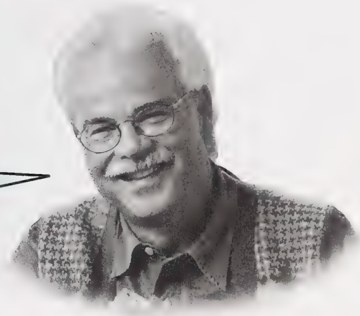
Usually, there is more than one way to get from one place to another. For example, to fly from one city to another, there may be a direct flight, a flight with one stopover at another city, one stopover at a different city, or two stopovers. These flight patterns are called travel networks.



Do you know that you can use matrices to analyse travel networks? In this activity, you will explore the role of matrices in solving a variety of **network problems**.

Turn to page 72 of the textbook and read the introductory paragraphs of Tutorial 2.3, "Solving Network Problems with Matrices."

To check your understanding of what you have read, cover the table and the corresponding matrix on the page. Can you reproduce the table and the matrix from the map at the top of the page without help?

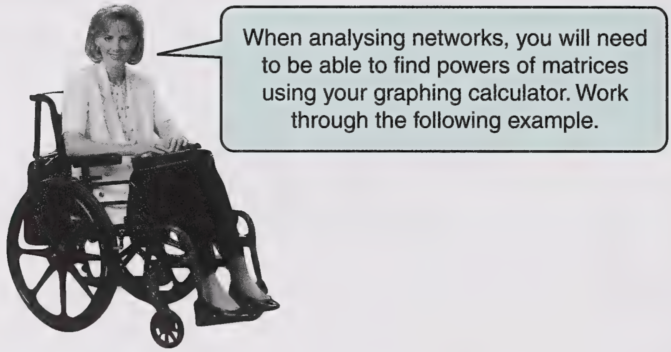


1. Complete exercises 1 to 7 of "Investigation: Determining the Number of Stopovers" on page 73 of the textbook. **Note:** A stopover may be any city on the route, not just a city that is between two other cities.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 75–76.

Turn to page 1 of Assignment Booklet 2B and answer question 1.

You may wish to contact your teacher for additional help with matrix operations.



Example

a. For a matrix, A, rewrite A^2 and A^3 as a product of matrices.

b. For matrix A, if A^2 exists, what is the relationship between its number of rows and its number of columns?

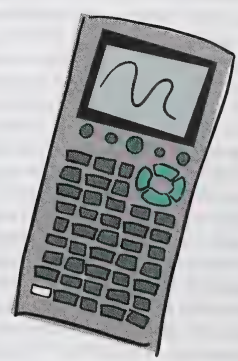
c. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, determine A^2 and A^3 using your graphing calculator.

Solution

a. $A^2 = A \times A$ $\quad\quad$ $A^3 = A^2 \times A$ or $A \times A^2$ or $A \times A \times A$

b. Recall that for matrices A and B, $A \times B$ exists if the number of columns of matrix A equals the number of rows of matrix B.

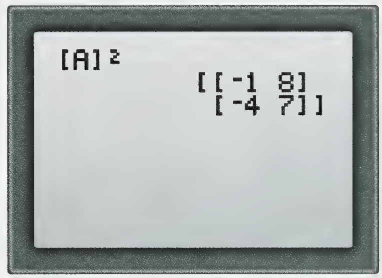
Because $A^2 = A \times A$, for A^2 to exist, the number of rows of matrix A and the number of columns of matrix A must be the same. Therefore, matrix A must be a square matrix for A^2 (or any power of A) to be defined.



c. Enter $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ into your graphing calculator.

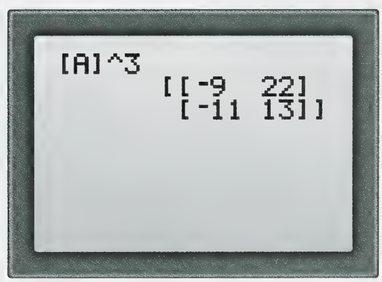
Determine A^2.

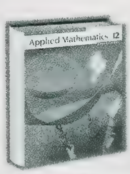
MATRX 1 (1:[A] 2×2) x^2
ENTER



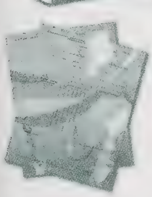
Determine A^3.

MATRX 1 (1:[A] 2×2) ^ 3
ENTER



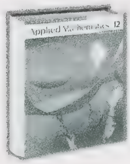


Not all network problems involve travel. Turn to pages 74 and 75 of the textbook and work through "Example: Create a Spy Network."



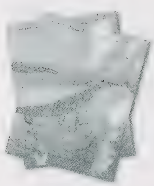
Turn to pages 1 and 2 of Assignment Booklet 2B
and answer question 2.

As you work through the following exercises, check your solution after each question for completeness and accuracy against the solution given in the Appendix. Use these worked solutions as models for each question you try. You will quickly enhance your understanding of solving network problems.



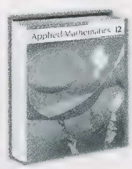
2. Turn to pages 76 to 79 of the textbook and answer exercises 1, 2.a., 5, 6, and 7 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 77–83.



Turn to pages 2 to 5 of Assignment Booklet 2B and answer questions 3, 4, and 5.

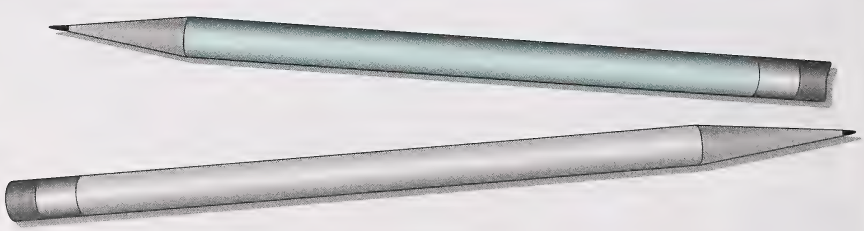
Looking Back



In this activity, you used matrices to model and solve network problems.

3. Turn to page 79 of the textbook and answer "Communicating the Ideas."

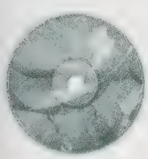
Compare your response with the suggested answer in the Appendix, Activity 3, pages 83–84.



ACTIVITY 4

Solving Transition Problems with Matrices

Have you noticed how loyal many people are to particular vehicle makes or models? Once they have purchased a particular car or truck, it is almost as though it becomes part of their personality. When they are in the market for a new vehicle, they are likely to replace their old car or truck with the same make or model. Manufacturers are very familiar with brand loyalty and make every effort, through advertising and incentives, to convince first-time buyers to purchase from them and not from their competitors.



For samples of the uses of matrices in business and government, you may wish to view the segments *Iterations and Matrices* and *Matrices* once again on the Applied Mathematics 30 CD.

Consider the following scenario:

> Suppose there are two car dealerships in your home town: Domestic Motors and Import Auto. Market research reveals that 60% of first-time buyers purchase their cars from Domestic Motors and 40% of first-time buyers purchase their cars from Import Auto. For people who purchased a vehicle from Domestic Motors, the probability that they will buy their next vehicle from Domestic Motors is 65% and the probability that they will buy their next vehicle from Import Auto is 35%. For people who purchased a vehicle from Import Auto, the probability that they will buy their next vehicle from Import Auto is 70% and the probability that they will buy their next vehicle from Domestic Auto is only 30%.

The probabilities for first-time buyers can be summarized in an **initial probability matrix**, P_0. In this row matrix, the respective probabilities for Domestic Motors (DM) and Import Auto (IA) are 0.60 and 0.40.

$$P_0 = \begin{array}{cc} \text{DM} & \text{IA} \\ \left[0.60\right. & \left.0.40\right] \end{array}$$

To calculate the percentages of people who purchase their next vehicle from these dealerships, you form a **transition matrix**, T, which will be used to transform the initial probability matrix, P_0.

$$T = \begin{array}{c} \begin{array}{cc} \text{DM} & \text{IA} \end{array} \\ \begin{bmatrix} 0.65 & 0.35 \\ 0.30 & 0.70 \end{bmatrix} \end{array}$$

The first row of this matrix consists of the probabilities for second-time buyers who purchased their first vehicle at Domestic Motors. The second row of this matrix consists of the probabilities for second-time buyers who purchased their first vehicle at Import Auto.

To find the percentages for all second-time buyers, evaluate $P_0 \times T$. Call this product P_1.

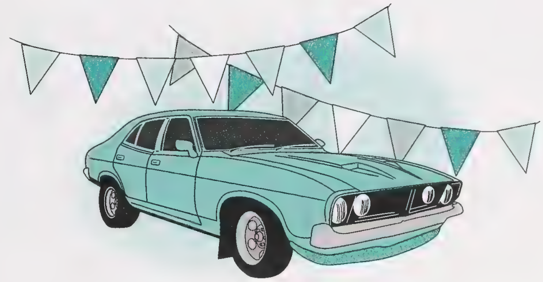
$$\begin{aligned} P_1 &= P_0 \times T \\ &= \begin{bmatrix} 0.60 & 0.40 \end{bmatrix} \times \begin{bmatrix} 0.65 & 0.35 \\ 0.30 & 0.70 \end{bmatrix} \\ &= \begin{bmatrix} (0.60 \times 0.65) + (0.40 \times 0.30) & (0.60 \times 0.35) + (0.40 \times 0.70) \end{bmatrix} \\ &= \begin{bmatrix} 0.39 + 0.12 & 0.21 + 0.28 \end{bmatrix} \\ &= \begin{bmatrix} 0.51 & 0.49 \end{bmatrix} \end{aligned}$$

Therefore, 51% of second-time buyers purchase their vehicles from Domestic Motors and 49% purchase their vehicles from Import Auto.

The preceding scenario is representative of what you will be doing in this activity. You will explore problem situations involving initial probability matrices and transition matrices.

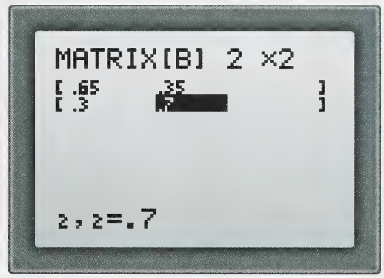
Example

Based on the scenario described at the beginning of this activity, use your graphing calculator and find the probabilities that buyers will purchase their third vehicle from Domestic Motors and Import Auto.



Solution

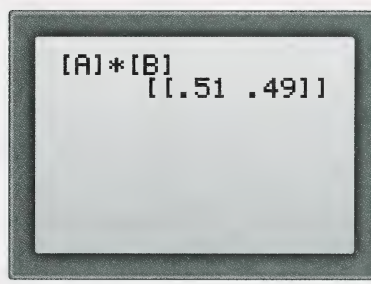
Enter P_0 as matrix A and matrix T as matrix B on your graphing calculator.



Method 1: Using P_1

Earlier, you found matrix P_1, the matrix for second-time buyers, by multiplying P_0 by T. Use your graphing calculator to check this matrix.

MATRX 1 (1:[A] 1×2) × MATRX 2 (2:[B] 2×2) ENTER



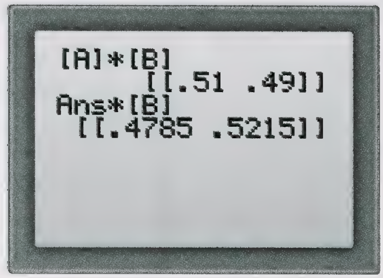
Let P_2 represent the probability matrix for third-time buyers. To determine P_2, multiply P_1 by the transition matrix, T.

$$
\begin{aligned}
P_2 &= P_1 \times T \\
&= \begin{bmatrix} 0.51 & 0.49 \end{bmatrix} \times \begin{bmatrix} 0.65 & 0.35 \\ 0.30 & 0.70 \end{bmatrix} \\
&= \begin{bmatrix} (0.51 \times 0.65) + (0.49 \times 0.30) & (0.51 \times 0.35) + (0.49 \times 0.70) \end{bmatrix} \\
&= \begin{bmatrix} 0.4785 & 0.5215 \end{bmatrix}
\end{aligned}
$$

Using your graphing calculator,

2nd [ANS] × MATRX 2

(2:[B] 2×2) ENTER



Therefore, 47.85% of third-time buyers purchase their vehicles from Domestic Motors and 52.15% purchase their vehicles from Import Auto.

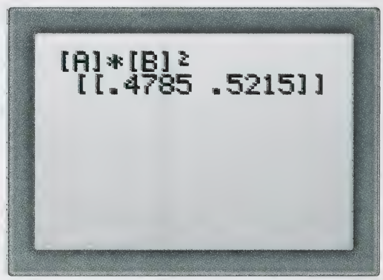
Method 2: Using P_0

$$\begin{aligned} P_2 &= P_1 \times T \\ &= \left(P_0 \times T\right) \times T \\ &= P_0 \times \left(T \times T\right) \\ &= P_0 \times T^2 \end{aligned}$$

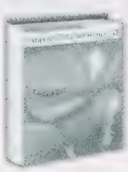
Therefore, to find the probabilities for third-time buyers, you must multiply the initial probability matrix, P_0, by the square of the transition matrix, T.

MATRX 1 (1:[A] 1×2) × MATRX

2 (2:[B] 2×2) ENTER



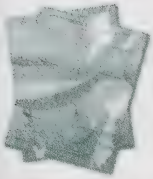
Therefore, 47.85% of third-time buyers purchase their vehicles from Domestic Motors and 52.15% purchase their vehicles from Import Auto.



Turn to page 80 of the textbook and read the introductory paragraphs of Tutorial 2.4, "Solving Transition Problems with Matrices."

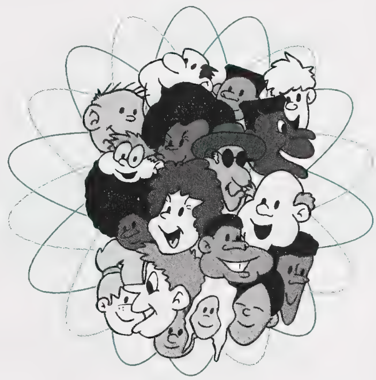
1. Complete exercises 1 to 7 of "Investigation: Finding n-Step Probabilities" on pages 80 and 81 of the textbook.

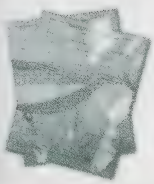
Compare your responses with the suggested answers in the Appendix, Activity 4, pages 85–86.



Turn to page 5 of Assignment Booklet 2B
and answer question 6.

Turn to page 81 of the textbook and carefully review the information in the coloured box preceding the example. Then work through "Example: Predict Population Shifts" on pages 81 and 82 of the textbook.





Turn to pages 6 and 7 of Assignment Booklet 2B
and answer question 7.

Example

Lynn has been working hard to get good grades. The probability of Lynn getting a good grade on an initial assignment is 85%. It seems that getting one good grade makes Lynn study harder, thus making the probability of getting a good grade twice in a row 95%. A poor grade hurts Lynn's confidence, thus making the probability of getting a poor grade twice in a row 30%. Build an initial probability matrix and a transition matrix; then determine the probability of Lynn getting a good grade on the tenth assignment. Round your answer to the nearest percent.

Solution

The probability of a good grade is 85%, thus making the probability of a poor grade $100\% - 85\% = 15\%$.

Therefore, the initial probability matrix is $P_0 = \begin{bmatrix} 0.85 & 0.15 \end{bmatrix}$.

Notice that the positive outcome is placed first in this case.
You could build the matrix in the other order if you wish.

The transition matrix must be built to match the initial probability matrix. Because the first element of P_0 is the probability of getting a good grade, the first row of the transition matrix should represent the probabilities after getting a good grade on the first assignment. This row would contain the values 95% and 5%. Because the second element of P_0 is the probability of getting a poor grade, the second row of the transition matrix should represent the probabilities resulting after a poor grade on the first assignment. This row would contain the values 70% and 30%.

Therefore, the transition matrix is

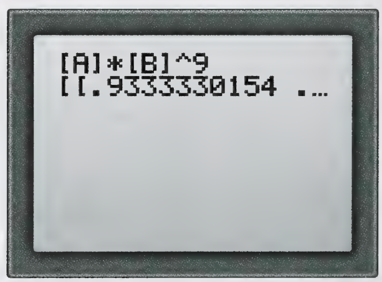
$$\begin{array}{cc} & \begin{array}{cc} \text{good grade} & \text{poor grade} \end{array} \\ T = & \begin{bmatrix} 0.95 & 0.05 \\ 0.70 & 0.30 \end{bmatrix} \begin{array}{l} \longleftarrow \text{a good grade on first assignment} \\ \longleftarrow \text{a poor grade on first assignment} \end{array} \end{array}$$

Notice that once it is decided that "good-grade" probabilities come first when building matrix P_0, the rows for the transition matrix also have the good-grade probabilities first. Another way of expressing this is to say that the first column of both P_0 and T contains the good-grade probabilities, and the second column contains the poor-grade probabilities.

Use your graphing calculator to find the probability of Lynn getting a good grade on the tenth assignment. Let matrix A represent P_0 and matrix B represent T.

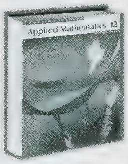
To find the probability of getting a good grade on the tenth assignment, determine the product of $P_0 \times T^9 \left(\text{or } A \times B^9\right)$.

MATRX 1 × MATRX 2 ^ 9 ENTER



$\therefore\ P_0 \times T^9 \doteq \begin{bmatrix} 0.93 & 0.07 \end{bmatrix}$

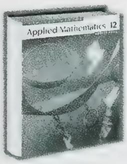
The probability that Lynn will get a good grade on the tenth assignment is about 93%.



2. Answer exercises 1, 2, and 3 of "Discussing the Ideas" on page 83 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 4, page 87.

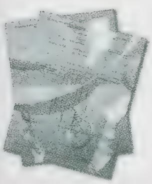
In the following exercises, you will use transition matrices to make predictions. In each exercise, ask yourself if your answer seems reasonable in the context of the problem situation. If your answer is not reasonable, you may have made an incorrect entry in your calculator.



3. Turn to pages 84 and 85 of the textbook and answer the following:

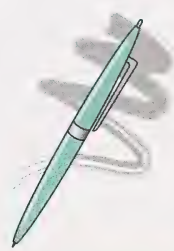
   a. exercises 5, 6, and 7 of "Exercises: Checking Your Skills"
   b. exercise 10 of "Exercises: Extending Your Thinking"

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 87–90.

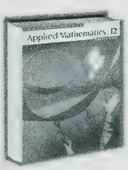


Turn to page 7 of Assignment Booklet 2B and answer question 8.

Looking Back

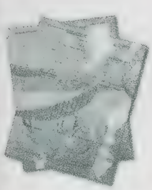


In this activity, you explored using initial probability matrices and transition matrices to make predictions in a variety of problem situations.



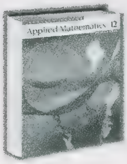
4. Turn to page 85 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 4, page 90.



Turn to pages 8 to 10 of Assignment Booklet 2B and answer questions 9 and 10.

Module Review



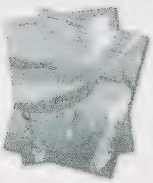
This module dealt with Chapter 2: Matrices in the *Addison-Wesley Applied Mathematics 12 Source Book.*

Turn to page 88 of the textbook and review the skills and concepts that were developed in this module. Also, read the important results and formulas that you discovered.

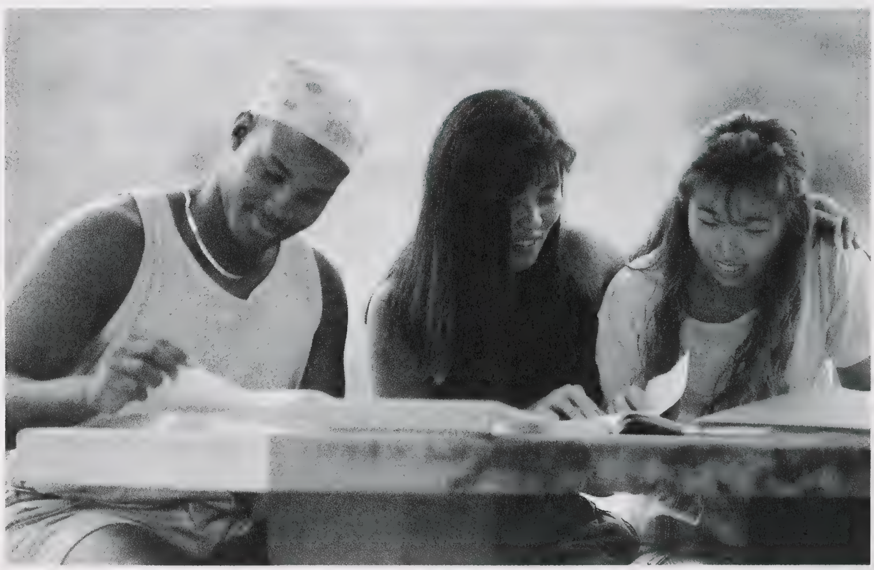
1. Answer exercise 1 of Part A of "What Should I Be Able to Do?" on page 89 of the textbook.

2. Answer exercises 2.b., 2.d, 5, 8, and 10 of Part B of "What Should I Be Able to Do?" on pages 90 to 92 of the textbook.

Compare your responses with the suggested answers in the Appendix, Module Review, pages 90–95.

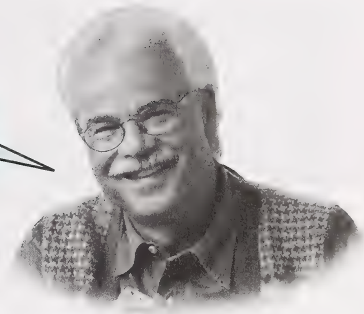


Turn to pages 11 to 18 of Assignment Booklet 2B and complete the Module Review Assignment.



If you had difficulties understanding the skills and concepts in Module 2: Matrices, it is recommended that you contact your teacher for some extra help activities. If you have a clear understanding of the skills and concepts in Matrices, you may wish to do the following enrichment activity. You may decide to do both.

Enrichment



An important application of matrices is solving systems of linear equations.

Consider the following system of equations:

$$3x + 2y = 9 \quad (1)$$
$$4x - 1y = 1 \quad (2)$$

First, form three matrices. Call the first matrix A. The elements of this matrix are the coefficients of x and y obtained from the two equations.

$$A = \begin{bmatrix} 3 & 2 \\ 4 & -1 \end{bmatrix}$$

Call the second matrix X. It is a column matrix obtained from the variables x and y.

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

Call the third matrix B. It is a column matrix obtained from the constants on the right side of each equation.

$$B = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$$

You can show that the matrix equation $AX = B$ represents the original system of equations.

$$AX = B$$

$$\begin{bmatrix} 3 & 2 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$$ ← Substitute for A, X, and B.

$$\begin{bmatrix} 3x & 2y \\ 4x & -1y \end{bmatrix} = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$$ ← Use the definition for matrix multiplication.

Because the two matrices are equal,

$$3x + 2y = 9$$
$$4x - 1y = 1$$

This is the system of equations you were given originally! Before solving the matrix equation $AX = B$, consider a more familiar equation: $ax = b$. Now, solve this equation for *x*.

$$\begin{aligned} ax &= b \\ \frac{ax}{a} &= \frac{b}{a} \\ x &= \frac{b}{a} \end{aligned}$$

You could also solve this equation by multiplying both sides by the multiplicative inverse, which is $\frac{1}{a}$ or a^{-1}.

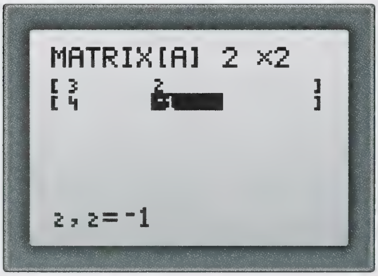
$$\begin{aligned} ax &= b \\ a^{-1}ax &= a^{-1}b \\ a^{0}x &= \left(\frac{1}{a}\right)b \\ 1x &= \frac{b}{a} \\ x &= \frac{b}{a} \end{aligned}$$

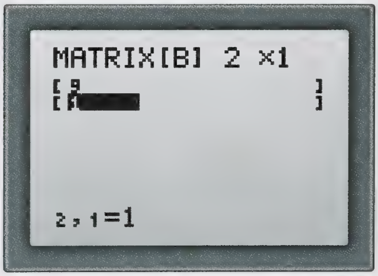
You can solve $AX = B$ in the same way.

$$\begin{aligned} AX &= B \\ A^{-1}(AX) &= A^{-1}(B) \\ X &= A^{-1}B \end{aligned}$$

Notice the order of multiplication here.

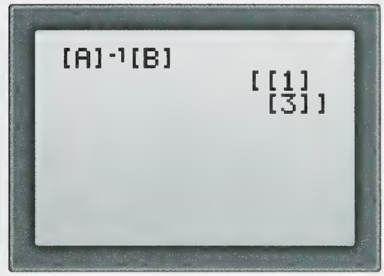
Now, you will use your graphing calculator to find *X*. First, enter matrices *A* and *B* into your graphing calculator.





Now, solve $X = A^{-1}B$.

MATRX 1 ([A] 2×2) x^{-1} MATRX 2 ([B] 2×1) ENTER



$\therefore\ X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

The solution to the system is $x = 1$ and $y = 3$.

Check the solution by substituting $x = 1$ and $y = 3$ into the original equations.

LS	RS
$3x+2y$	9
$=3(1)+2(3)$	
$=3+6$	
$=9$	

LS = RS

LS	RS
$4x-1y$	1
$=4(1)-1(3)$	
$=4-3$	
$=1$	

LS = RS

The solution $x = 1$ and $y = 3$ is correct.

The next example involves three equations in three unknowns.

Example

Use matrices to solve the following system of equations:

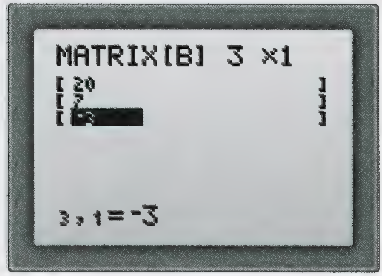
$$x+2y-5z=20 \quad (1)$$
$$2x+y-z=7 \quad (2)$$
$$x+y+2z=-3 \quad (3)$$

Solution

Form matrices A, X, and B.

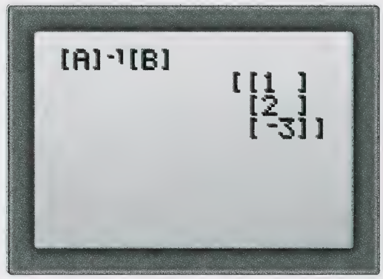
$$A = \begin{bmatrix} 1 & 2 & -5 \\ 2 & 1 & -1 \\ 1 & 1 & 2 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad B = \begin{bmatrix} 20 \\ 7 \\ -3 \end{bmatrix}$$

Enter matrix A and matrix B into your graphing calculator.



$$AX = B$$
$$X = A^{-1}B$$

Solve $X = A^{-1}B$.



$\boxed{\text{MATRX}}$ $\boxed{1}$ ([A] 3×3) $\boxed{x^{-1}}$ $\boxed{\text{MATRX}}$

$\boxed{2}$ ([B] 3×1) $\boxed{\text{ENTER}}$

$\therefore\ x = 1,\ y = 2,$ and $z = -3$

1. Check the answers given in the preceding example by substituting them into the original equations.

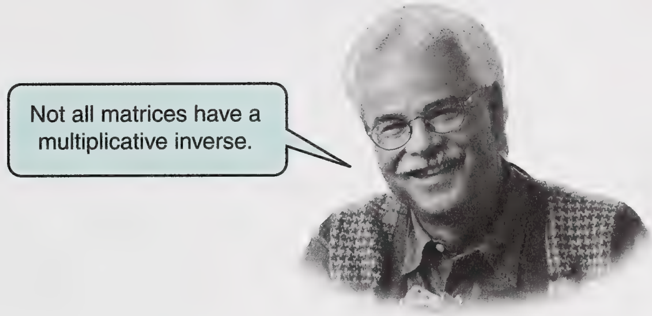
2. Solve and check the following system of equations:

$x + 3y = 5$ (1)
$2x - y = -4$ (2)

3. Solve and check the following system of equations:

$w + x + y + z = 4$ (1)
$w + 3y + 5z = 9$ (2)
$w + z = 2$ (3)
$x - y - z = -1$ (4)

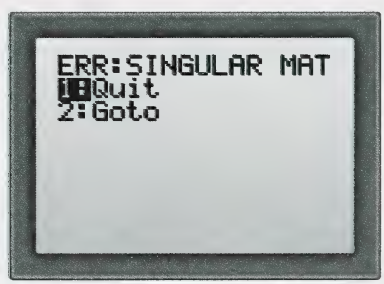
Compare your responses with the suggested answers in the Appendix, Module Review: Enrichment, pages 96–97.



For example, enter the matrix $A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$ into your graphing calculator. (It relates to the pair of equations $6x - 3y = -6$ and $-2x + y = -2$.)

Now, try to find A^{-1}.

MATRX 1 (1:[A] 2×2) x^{-1} ENTER



This tells you that matrix A is a singular matrix; its inverse, A^{-1}, does not exist.

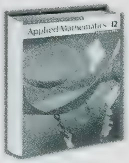
Completing the Project

By now, you should have completed the initial research for your module project, Creating a Long-Distance Telephone Plan. You should have a good understanding of the various options offered by competing long-distance telephone providers in your area.

For this project, you will be asked to complete two parts. First, you will examine which provider in your area best meets your family's needs, and you will propose a competing plan. Second, you will survey people in your area to predict how many would convert to your plan.

Turn to pages 68 to 70 of the textbook and read the information given. Then complete exercises 1 to 6 of "Comparing the Competition." Keep a copy of your responses to these exercises in the project section of your mathematics binder. You will need these responses to help you complete the questions for your project.

Before you continue your module project, look at the sample project in Part C of "What Should I Be Able to Do?" on pages 93 and 94 of the textbook.

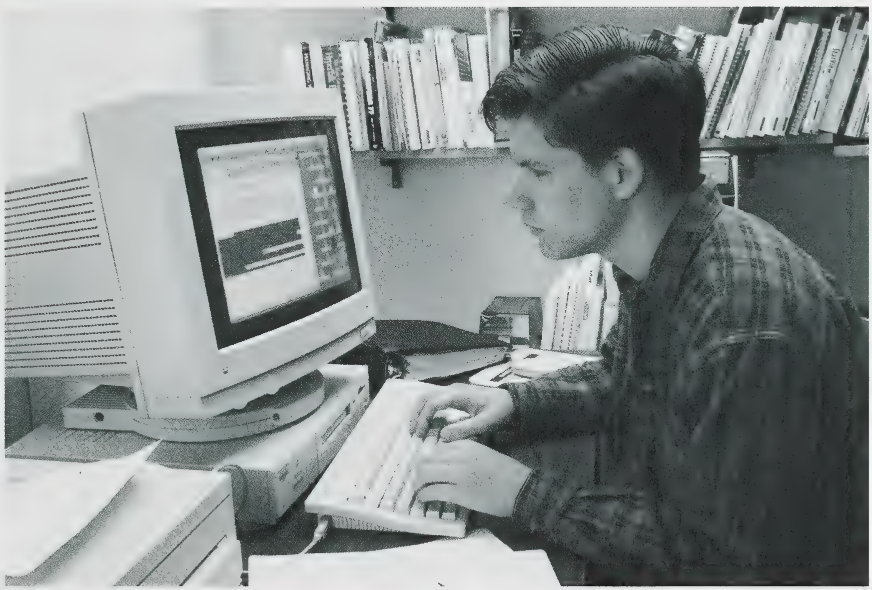


1. Answer exercise 11 of Part C of "What Should I Be Able to Do?" on page 93 of the textbook.

Keep a copy of your responses in the project section of your mathematics binder. You will need this information to answer the questions dealing with your family's long-distance needs in the project you submit.

2. Answer exercise 12 of Part C of "What Should I Be Able to Do?" on page 94 of the textbook.

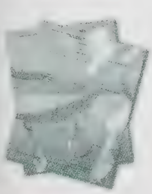
Compare your responses with the suggested answers in the Appendix, Module Project, page 98.



Module Project

Now that you have more insight into the module project, revise your answers. Then complete the Module 2 project, Creating a Long-Distance Telephone Plan.

You may use your responses from the textbook exercises on pages 50, 68 to 70, and 93 and 94 to help you complete the project. (These responses should be in the project section of your mathematics binder.)



Turn to pages 19 to 23 of Assignment Booklet 2B and complete the module project.

MODULE SUMMARY

In this module, with the help of technology, you explored matrices and matrix operations. You learned how to add and subtract matrices, multiply a matrix by a scalar, as well as multiply pairs of matrices. You modelled and solved problems that involved these operations. In particular, you modelled and solved network problems and transition problems. In your module project, you used matrices to critique long-distance telephone plans and propose a plan of your own.

Well before long-distance telephone calls and e-mail, people relied on the post office to communicate with friends and business colleagues. Planning and operating efficient mail-delivery systems are examples of complex network problems. These are not modern problems by any means. Did you know that the ancient Egyptians used relay systems to speed up mail delivery? Did you know that the first-century Romans had next-day delivery within a 250-km radius of Rome? With modern technology and mathematical discoveries, solving these kinds of problems is becoming simpler.

APPENDIX

Glossary
Calculator Functions
Suggested Answers
Image Credits

Glossary

column matrix: a matrix with only a single column

dimensions of a matrix: the number of rows, m, by the number of columns, n, in a matrix, generally written as $m \times n$

element: an entry in a matrix

initial probability matrix: a matrix consisting of the distribution of probabilities for the first stage of a multistep problem

matrix: a rectangular array of elements

network problem: a problem involving possible routes between destinations

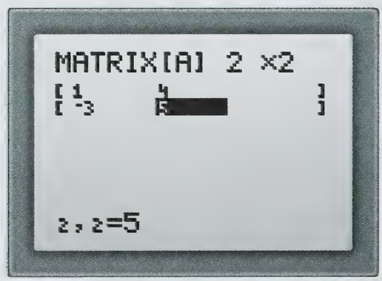
row matrix: a matrix with only a single row

scalar: a constant

square matrix: a matrix with the same number of rows and columns

transition matrix: a square matrix showing how the probability of an event is dependent on the probability of another event occurring

Calculator Functions

Function	Keystrokes	Example
Entering a matrix	MATRX ◀ 1  	2 ENTER 2 ENTER 1 ENTER 4 ENTER (–) 3 ENTER 5 ENTER 

Deleting a matrix	2nd [MEM] 2 (Delete…) 5 (Matrix…) DELETE:Matrix ▸[A] 44 [B] 44 [C] 44	ENTER ENTER ENTER DELETE:Matrix
Storing the results of a matrix multiplication into another matrix	MATRX 1 × MATRX 2 STO➡ MATRX 3 [A]*[B]→[C]█	ENTER [A]*[B]→[C] [[62 5] [50 17]]

Suggested Answers

Activity 1: Modelling Problems with Matrices

1. Textbook exercises 1, 2, and 3 of "Discussing the Ideas," p. 55

1. The elements of a matrix are enclosed within brackets to indicate that the matrix is a single entity comprised of elements.

2. The notation a_{ij} is useful in communicating the row and column in which the element is located. The order of the subscripts, i and j, is important. Subscript i indicates the row, and subscript j indicates the column. The order of the subscripts indicates the row and then the column. For example, a_{23} is the element in row 2, column 3.

3. The location of an element in a matrix and the location of a cell in a spreadsheet are both specified by row and column. For a matrix, each element's location is designated by two subscripts. The first subscript indicates the row, and the second subscript indicates the column.

$$a_{ij}$$

row column

Matrices and spreadsheets differ in that the columns of a spreadsheet are represented by letters. Also, the column in a spreadsheet is given first and the row is given second.

cell B3

column row

2. Textbook exercises 1.a., 1.b., and 3 of "Exercises: Checking Your Skills," pp. 55 and 56

1. a. $$A+C=\begin{bmatrix} 3 & -2 & 6 \\ 8 & 1 & 0 \end{bmatrix}+\begin{bmatrix} 1 & 10 & -3 \\ 4 & 6 & -2 \end{bmatrix}$$

$$=\begin{bmatrix} 3+1 & -2+10 & 6+(-3) \\ 8+4 & 1+6 & 0+(-2) \end{bmatrix}$$

$$=\begin{bmatrix} 4 & 8 & 3 \\ 12 & 7 & -2 \end{bmatrix}$$

b. $B+C$ is undefined because the dimensions of B and C are not the same. The dimensions of B are 3×1, and the dimensions of C are 2×3.

3. Let A represent the Against Eastern Teams table and B represent the Against Western Teams table.

$$A=\begin{bmatrix} 1 & 0 & 2 & 2 \\ 2 & 0 & 0 & 4 \\ 1 & 0 & 1 & 2 \\ 1 & 0 & 2 & 2 \end{bmatrix} \qquad B=\begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 0 & 2 & 6 \\ 1 & 0 & 4 & 2 \\ 1 & 0 & 3 & 2 \end{bmatrix}$$

Notice how the matrix elements match the numbers in the tables.

$$\therefore\ A+B=\begin{bmatrix}1&0&2&2\\2&0&0&4\\1&0&1&2\\1&0&2&2\end{bmatrix}+\begin{bmatrix}4&0&0&8\\3&0&2&6\\1&0&4&2\\1&0&3&2\end{bmatrix}$$

$$=\begin{bmatrix}1+4&0+0&2+0&2+8\\2+3&0+0&0+2&4+6\\1+1&0+0&1+4&2+2\\1+1&0+0&2+3&2+2\end{bmatrix}$$

$$=\begin{bmatrix}5&0&2&10\\5&0&2&10\\2&0&5&4\\2&0&5&4\end{bmatrix}$$

The following table shows the combined standings.

	W	T	L	Points
BC	5	0	2	10
Calgary	5	0	2	10
Saskatchewan	2	0	5	4
Edmonton	2	0	5	4

3. Textbook exercises 1.c., 1.d., 2, 5.a., 5.b., 6, and 7 of "Exercises: Checking Your Skills," pp. 55 to 57

1. c. $3B=3\begin{bmatrix}3\\2\\-5\end{bmatrix}$

$$=\begin{bmatrix}3(3)\\3(2)\\3(-5)\end{bmatrix}$$

$$=\begin{bmatrix}9\\6\\-15\end{bmatrix}$$

d. $2A-C=2\begin{bmatrix}3&-2&6\\8&1&0\end{bmatrix}-\begin{bmatrix}1&10&-3\\4&6&-2\end{bmatrix}$

$$=\begin{bmatrix}2(3)&2(-2)&2(6)\\2(8)&2(1)&2(0)\end{bmatrix}-\begin{bmatrix}1&10&-3\\4&6&-2\end{bmatrix}$$

$$=\begin{bmatrix}6&-4&12\\16&2&0\end{bmatrix}-\begin{bmatrix}1&10&-3\\4&6&-2\end{bmatrix}$$

$$=\begin{bmatrix}6-1&-4-10&12-(-3)\\16-4&2-6&0-(-2)\end{bmatrix}$$

$$=\begin{bmatrix}5&-14&15\\12&-4&2\end{bmatrix}$$

2. **a.** $$\begin{bmatrix} 3+2 & 8+2 & 1+2 \\ 2+2 & 4+2 & 6+2 \\ 7+2 & 0+2 & 5+2 \end{bmatrix} = \begin{bmatrix} 5 & 10 & 3 \\ 4 & 6 & 8 \\ 9 & 2 & 7 \end{bmatrix}$$

The result is a magic square. All rows, columns, and diagonals add up to 18.

b. $$\begin{aligned} 2A &= 2\begin{bmatrix} 3 & 8 & 1 \\ 2 & 4 & 6 \\ 7 & 0 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 2(3) & 2(8) & 2(1) \\ 2(2) & 2(4) & 2(6) \\ 2(7) & 2(0) & 2(5) \end{bmatrix} \\ &= \begin{bmatrix} 6 & 16 & 2 \\ 4 & 8 & 12 \\ 14 & 0 & 10 \end{bmatrix} \end{aligned}$$

The result is a magic square. All rows, columns, and diagonals add up to 24.

c. $$\begin{aligned} A+\begin{bmatrix} 5 & 10 & 3 \\ 4 & 6 & 8 \\ 9 & 2 & 7 \end{bmatrix} &= \begin{bmatrix} 3 & 8 & 1 \\ 2 & 4 & 6 \\ 7 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 5 & 10 & 3 \\ 4 & 6 & 8 \\ 9 & 2 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 3+5 & 8+10 & 1+3 \\ 2+4 & 4+6 & 6+8 \\ 7+9 & 0+2 & 5+7 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 18 & 4 \\ 6 & 10 & 14 \\ 16 & 2 & 12 \end{bmatrix} \end{aligned}$$

The result is a magic square. All rows, columns, and diagonals add up to 30.

5. **a.** Answers will vary because exchange rates vary day to day. A sample answer is given.

   i. Can \$1.00 = US \$0.635

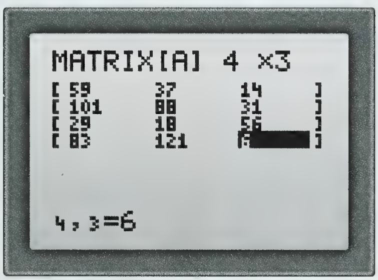
   ii. Can \$1.00 = ¥79.172 ¥ is the symbol for yen.

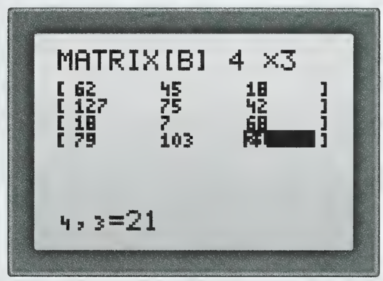
   b. In Canadian dollars, the row matrix that represents the cost of the four tours that start in Toronto is $\begin{bmatrix} 2240 & 2645 & 3050 & 3725 \end{bmatrix}$.

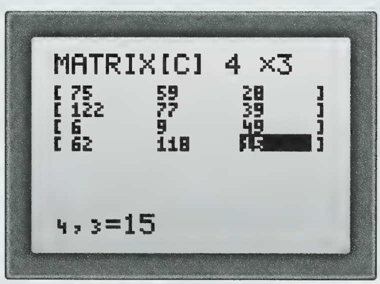
   Multiply this matrix by 0.635 to convert the costs to U.S. dollars.

$$
\begin{aligned}
\text{US\$} &= 0.635\begin{bmatrix} 2240 & 2645 & 3050 & 3725 \end{bmatrix} \\
&= \begin{bmatrix} 0.635(2240) & 0.635(2645) & 0.635(3050) & 0.635(3725) \end{bmatrix} \\
&= \begin{bmatrix} 1422.40 & 1679.58 & 1936.75 & 2365.38 \end{bmatrix}
\end{aligned}
$$

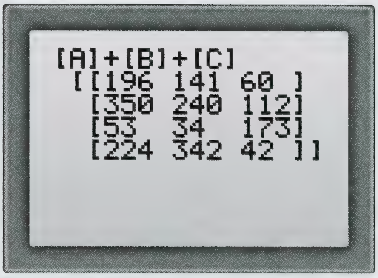
6. **a.** Enter the matrices *A*, *B*, and *C* into your graphing calculator. **Remember:** Clear any matrices on your calculator first.







b. MATRX 1 (1:[A] 4×3) + MATRX 2 (2:[B] 4×3) + MATRX 3 (3:[C] 4×3) ENTER

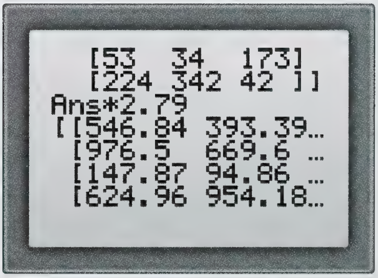


The total number of hours each client used each service is summarized in the following table.

	[illegible]	InfoSearch	[illegible]
Client A	196	141	60
Client B	350	240	112
Client C	53	34	173
Client D	224	342	42

c. Multiply the matrix obtained in exercise 6.b. by 2.79.

× 2 • 7 9 ENTER



Note: Because this calculation immediately follows the calculation in exercise 6.b., you only need to enter "×2.79" to perform the multiplication. When you start a new line with an operation sign, the calculator assumes you want to include the preceding answer in the calculation.

The total changes for each client for each service is summarized in the following table.

	[illegible]	InfoSearch	[illegible]
Client A	546.84	393.39	167.40
Client B	976.50	669.60	312.48
Client C	147.87	94.86	482.67
Client D	624.96	954.18	117.18

7. a. Let A represent the matrix for regular hours and B represent the matrix for overtime hours.

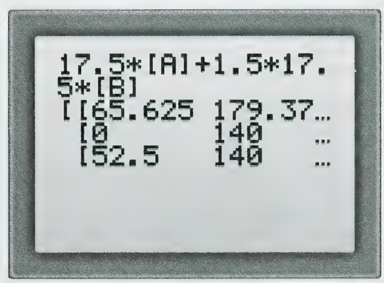
$$A = \begin{bmatrix} 3.75 & 8.00 & 8.00 & 8.00 & 8.00 & 8.00 & 8.00 \\ 0.00 & 8.00 & 8.00 & 8.00 & 8.00 & 8.00 & 6.50 \\ 3.00 & 8.00 & 8.00 & 8.00 & 8.00 & 8.00 & 8.00 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.00 & 1.50 & 0.00 & 0.00 & 1.00 & 1.75 & 2.00 \\ 0.00 & 0.00 & 1.00 & 1.00 & 0.75 & 1.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.50 & 0.50 & 0.50 \end{bmatrix}$$

Enter each matrix into your graphing calculator.

b. To determine the daily pay, calculate $17.50A + 1.5 \times 17.50B$.

1 7 • 5 × MATRX 1 (1:[A] 3×7)
\+ 1 • 5 × 1 7 • 5 ×
MATRX 2 (2:[B] 3×7) ENTER



Enter the values from your matrix into a table showing the daily pay for each worker. Round these values to the nearest cent.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
M. Cairns	$65.63	$179.38	$140.00	$140.00	$166.25	$185.94	$192.50
D. Gieselman	$0.00	$140.00	$166.25	$166.25	$159.69	$166.25	$113.75
S. Sahota	$52.50	$140.00	$140.00	$140.00	$153.13	$153.13	$153.13

4. Textbook exercise "Communicating the Ideas," p. 59

Answers will vary. A sample answer is given.

Matrices are rectangular arrays of elements (usually numbers). They can be added and subtracted by adding or subtracting corresponding elements. For example,

$$\begin{aligned}\begin{bmatrix}1 & 3\\ 5 & 7\end{bmatrix}+\begin{bmatrix}15 & 13\\ 11 & 9\end{bmatrix}&=\begin{bmatrix}1+15 & 3+13\\ 5+11 & 7+9\end{bmatrix}\\ &=\begin{bmatrix}16 & 16\\ 16 & 16\end{bmatrix}\end{aligned}$$

Matrices can be multiplied by a scalar (a constant number). For example,

$$\begin{aligned}3\times\begin{bmatrix}1 & 2 & 3\end{bmatrix}&=\begin{bmatrix}3(1) & 3(2) & 3(3)\end{bmatrix}\\ &=\begin{bmatrix}3 & 6 & 9\end{bmatrix}\end{aligned}$$

Matrices can be used to help solve consumer problems, such as time-and-a-half wages earned in a given situation, as well as inventory control and service usage over a given period.

Activity 2: Matrix Multiplication

1. Textbook exercises 1 to 6 of "Investigation 1: Multiplying Matrices," p. 60

1.

Team	[illegible]	Tie	[illegible]	Total
Blazers	$2\times7=14$	$1\times2=2$	$0\times4=0$	$14+2=16$
Hawks	$2\times5=10$	$1\times1=1$	$0\times7=0$	$10+1=11$
Tigers	$2\times8=16$	$1\times0=0$	$0\times5=0$	$16+0=16$
Eagles	$2\times4=8$	$1\times1=1$	$0\times8=0$	$8+1=9$

2. The Blazers' total points, 16, were determined by adding the 2 points from 2 ties to the 14 points from 7 wins.

The same method was used for the other three teams.

3. $S = \begin{bmatrix} 7 & 2 & 4 \\ 5 & 1 & 7 \\ 8 & 0 & 5 \\ 4 & 1 & 8 \end{bmatrix}$

4. $P = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

5. $T = \begin{bmatrix} 16 \\ 11 \\ 16 \\ 9 \end{bmatrix}$

6. $$S \times P = \begin{bmatrix} 7 & 2 & 4 \\ 5 & 1 & 7 \\ 8 & 0 & 5 \\ 4 & 1 & 8 \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} (7\times2)+(2\times1)+(4\times0) \\ (5\times2)+(1\times1)+(7\times0) \\ (8\times2)+(0\times1)+(5\times0) \\ (4\times2)+(1\times1)+(8\times0) \end{bmatrix}$$
$$= \begin{bmatrix} 14+2+0 \\ 10+1+0 \\ 16+0+0 \\ 8+1+0 \end{bmatrix}$$
$$= \begin{bmatrix} 16 \\ 11 \\ 16 \\ 9 \end{bmatrix}$$

2. Textbook exercises 1, 2, 3, and 5 of "Investigation 2: Dimensions of the Product Matrix," pp. 61 and 62

1. $S = \begin{bmatrix} 0 & 14 & 103 & 32 \\ 67 & 29 & 45 & 18 \\ 0 & 0 & 86 & 54 \end{bmatrix}$

2. $A = \begin{bmatrix} 5 \\ 6 \\ 8 \\ 6 \end{bmatrix}$

3. $$S \times A = \begin{bmatrix} 0 & 14 & 103 & 32 \\ 67 & 29 & 45 & 18 \\ 0 & 0 & 86 & 54 \end{bmatrix} \times \begin{bmatrix} 5 \\ 6 \\ 8 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} (0\times5)+(14\times6)+(103\times8)+(32\times6) \\ (67\times5)+(29\times6)+(45\times8)+(18\times6) \\ (0\times5)+(0\times6)+(86\times8)+(54\times6) \end{bmatrix}$$

$$= \begin{bmatrix} 1100 \\ 977 \\ 1012 \end{bmatrix}$$

The dimensions of $S \times A$ are 3×1. The number of rows in $S \times A$ is the same as the number of rows in S, and the number of columns in $S \times A$ is the same as the number of columns in A.

5. $$A \times S = \begin{bmatrix} 5 \\ 6 \\ 8 \\ 6 \end{bmatrix} \times \begin{bmatrix} 0 & 14 & 103 & 32 \\ 67 & 29 & 45 & 18 \\ 0 & 0 & 86 & 54 \end{bmatrix}$$

$$= \begin{bmatrix} (5\times0)+(\boxed{}\times67)+(\boxed{}\times0) & (5\times14)+(\boxed{}\times29)+(\boxed{}\times0)\ldots \\ (6\times0)+(\boxed{}\times67)+(\boxed{}\times0) & (6\times14)+(\boxed{}\times29)+(\boxed{}\times0)\ldots \\ (8\times0)+(\boxed{}\times67)+(\boxed{}\times0) & (8\times14)+(\boxed{}\times29)+(\boxed{}\times0)\ldots \\ (6\times0)+(\boxed{}\times67)+(\boxed{}\times0) & (6\times14)+(\boxed{}\times29)+(\boxed{}\times0)\ldots \end{bmatrix}$$

There is no number to place here.

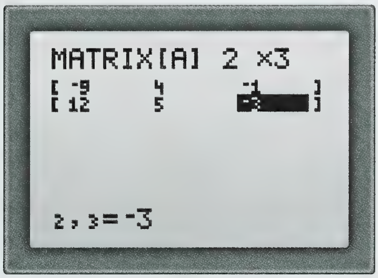
There are not enough columns in A to allow the process of multiplying $A \times S$; so, $A \times S$ is undefined.

3. Textbook exercises 1 and 2 of "Exercises: Checking Your Skills," p. 64

1. a. Let A and B represent the matrices.

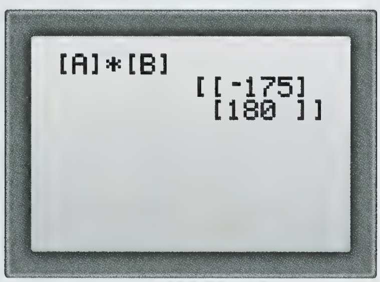
$$A=\begin{bmatrix}-9 & 4 & -1\\ 12 & 5 & -3\end{bmatrix} \qquad B=\begin{bmatrix}17\\ -6\\ -2\end{bmatrix}$$

Enter these matrices into your graphing calculator.



Determine $A \times B$.

MATRX 1 (1:[A] 2×3) × MATRX 2 (2:[B] 3×1) ENTER

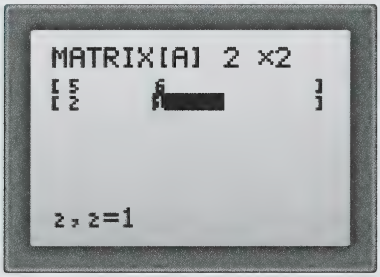


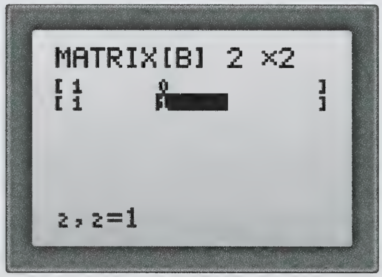
$$\therefore\ A\times B=\begin{bmatrix}-175\\ 180\end{bmatrix}$$

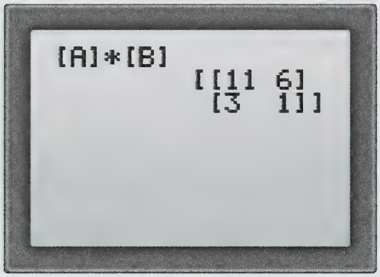
b. Let A and B represent the matrices.

$$A=\begin{bmatrix}5 & 6\\ 2 & 1\end{bmatrix} \qquad B=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$$

Enter the matrices, and determine $A \times B$.



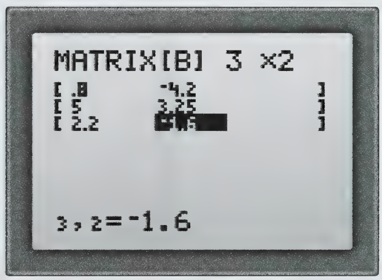


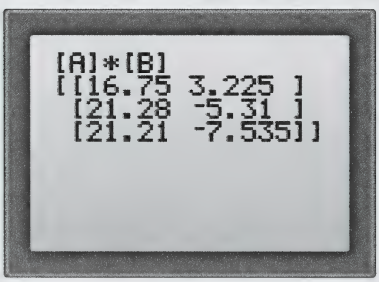


$$\therefore\ A \times B = \begin{bmatrix} 11 & 6 \\ 3 & 1 \end{bmatrix}$$

2. Enter matrices A and B into your graphing calculator.

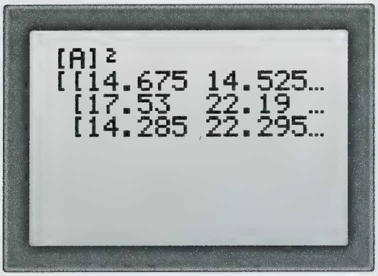
a.





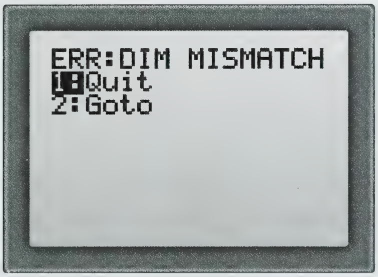
$$\therefore\ A \times B = \begin{bmatrix} 16.75 & 3.225 \\ 21.28 & -5.31 \\ 21.21 & -7.535 \end{bmatrix}$$

b.



$$\therefore A^2 = \begin{bmatrix} 14.675 & 14.525 & 6.5125 \\ 17.53 & 22.19 & 11.645 \\ 14.285 & 22.295 & 13.5375 \end{bmatrix}$$

c.



$B \times A$ is undefined because the number of columns in matrix B is not the same as the number of rows in matrix A.

4. Textbook exercises 3, 5, 7, and 9 of "Exercises: Checking Your Skills," pp. 64 to 67

3. a. Let matrix A represent the store's sales for the winter months.

$$A = \begin{bmatrix} 43 & 14 & 32 \\ 76 & 26 & 59 \\ 52 & 19 & 51 \end{bmatrix}$$

b. Let matrix B represent the profits on each item.

$$B = \begin{bmatrix} 78 \\ 47 \\ 65 \end{bmatrix}$$

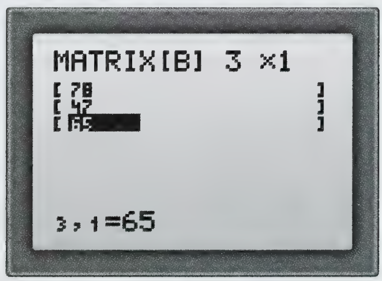
c. Method 1: Using Pencil and Paper

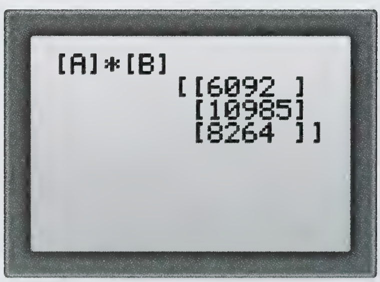
$$
\begin{aligned}
A \times B &= \begin{bmatrix} 43 & 14 & 32 \\ 76 & 26 & 59 \\ 52 & 19 & 51 \end{bmatrix} \times \begin{bmatrix} 78 \\ 47 \\ 65 \end{bmatrix} \\
&= \begin{bmatrix} (43 \times 78) + (14 \times 47) + (32 \times 65) \\ (76 \times 78) + (26 \times 47) + (59 \times 65) \\ (52 \times 78) + (19 \times 47) + (51 \times 65) \end{bmatrix} \\
&= \begin{bmatrix} 3354 + 658 + 2080 \\ 5928 + 1222 + 3835 \\ 4056 + 893 + 3315 \end{bmatrix} \\
&= \begin{bmatrix} 6092 \\ 10\,985 \\ 8264 \end{bmatrix}
\end{aligned}
$$

The monthy profits were \$6092 in November, \$10 985 in December, and \$8264 in January.

Method 2: Using a Graphing Calculator

Enter A and B into your graphing calculator, and determine $A \times B$.





The monthly profits were \$6092 in November, \$10 985 in December, and \$8264 in January.

5. a. Tigers: $\text{Points} = (30\times2)+(2\times1)+(8\times1)+(2\times0)$
$= 70$

Irish: $\text{Points} = (24\times2)+(9\times1)+(2\times1)+(7\times0)$
$= 59$

Colts: $\text{Points} = (25\times2)+(7\times1)+(0\times1)+(10\times0)$
$= 57$

Jets: $\text{Points} = (26\times2)+(1\times1)+(10\times1)+(5\times0)$
$= 63$

b. Let matrix A represent the standings and matrix B represent the points earned for a win, a tie with goals scored, a tie with no goals scored, and a loss.

$$A = \begin{bmatrix} 30 & 2 & 8 & 2 \\ 24 & 9 & 2 & 7 \\ 25 & 7 & 0 & 10 \\ 26 & 1 & 10 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Method 1: Using Paper and Pencil

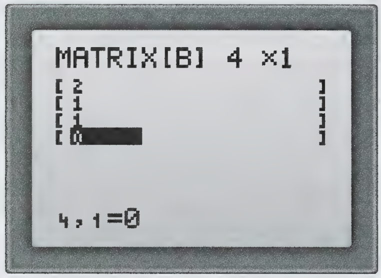
$$A\times B = \begin{bmatrix} 30 & 2 & 8 & 2 \\ 24 & 9 & 2 & 7 \\ 25 & 7 & 0 & 10 \\ 26 & 1 & 10 & 5 \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

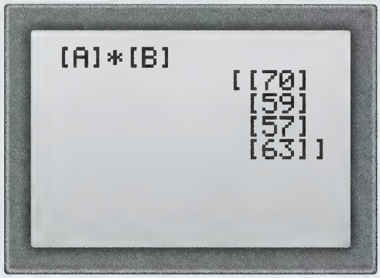
$$= \begin{bmatrix} (30\times2)+(2\times1)+(8\times1)+(2\times0) \\ (24\times2)+(9\times1)+(2\times1)+(7\times0) \\ (25\times2)+(7\times1)+(0\times1)+(10\times0) \\ (26\times2)+(1\times1)+(10\times1)+(5\times0) \end{bmatrix}$$

$$= \begin{bmatrix} 70 \\ 59 \\ 57 \\ 63 \end{bmatrix}$$

Method 2: Using a Graphing Calculator

Enter matrices A and B into your calculator, and determine $A \times B$.





c. Let matrix C represent point system i., matrix D represent point system ii., and matrix E represent point system iii.

$$C = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix} \qquad D = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad E = \begin{bmatrix} 5 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

Method 1: Pencil and Paper

$$A \times C = \begin{bmatrix} 30 & 2 & 8 & 2 \\ 24 & 9 & 2 & 7 \\ 25 & 7 & 0 & 10 \\ 26 & 1 & 10 & 5 \end{bmatrix} \times \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} (30\times3)+(2\times1)+(8\times1)+(2\times0) \\ (24\times3)+(9\times1)+(2\times1)+(7\times0) \\ (25\times3)+(7\times1)+(0\times1)+(10\times0) \\ (26\times3)+(1\times1)+(10\times1)+(5\times0) \end{bmatrix}$$

$$= \begin{bmatrix} 100 \\ 83 \\ 82 \\ 89 \end{bmatrix}$$

$$A \times D = \begin{bmatrix} 30 & 2 & 8 & 2 \\ 24 & 9 & 2 & 7 \\ 25 & 7 & 0 & 10 \\ 26 & 1 & 10 & 5 \end{bmatrix} \times \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} (30\times3)+(2\times1)+(8\times0)+(2\times0) \\ (24\times3)+(9\times1)+(2\times0)+(7\times0) \\ (25\times3)+(7\times1)+(0\times0)+(10\times0) \\ (26\times3)+(1\times1)+(10\times0)+(5\times0) \end{bmatrix}$$

$$= \begin{bmatrix} 92 \\ 81 \\ 82 \\ 79 \end{bmatrix}$$

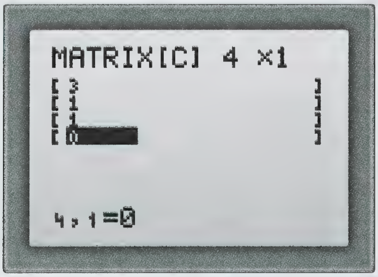
$$A \times E = \begin{bmatrix} 30 & 2 & 8 & 2 \\ 24 & 9 & 2 & 7 \\ 25 & 7 & 0 & 10 \\ 26 & 1 & 10 & 5 \end{bmatrix} \times \begin{bmatrix} 5 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

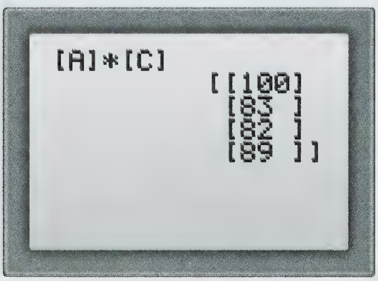
$$= \begin{bmatrix} (30\times5)+(2\times3)+(8\times0)+(2\times0) \\ (24\times5)+(9\times3)+(2\times0)+(7\times0) \\ (25\times5)+(7\times3)+(0\times0)+(10\times0) \\ (26\times5)+(1\times3)+(10\times0)+(5\times0) \end{bmatrix}$$

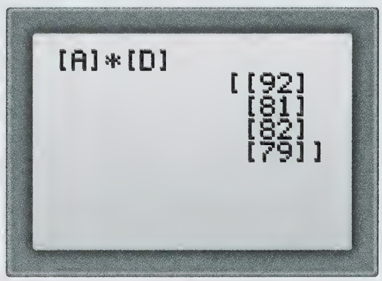
$$= \begin{bmatrix} 156 \\ 147 \\ 146 \\ 133 \end{bmatrix}$$

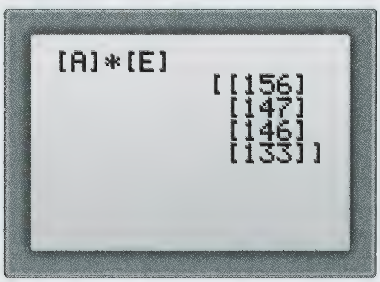
Method 2: Using a Graphing Calculator

Enter matrices C, D, and E into your calculator, and determine $A \times C$, $A \times D$, and $A \times E$.









d. Point system iii. will make the Irish second in the standings.

e. Point system ii. will make the Colts second in the standings.

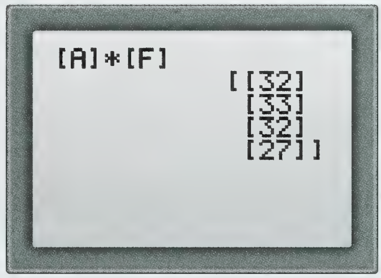
f. Point system i. will make the Jets second in the standings.

g. Answers will vary. A sample answer is given.

Make both a win and a tie with goals scored worth 1 point each, and make both a tie with no goals scored and a loss worth 0 points each. Let matrix F represent this point system.

$$\therefore\ F = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Determine $A \times F$ using your graphing calculator.

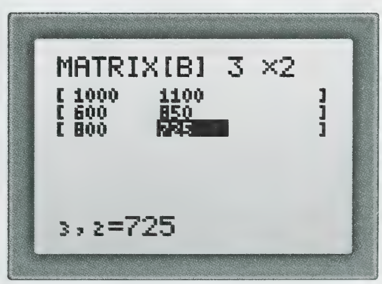


With this point system, the Irish are in first place, not the Tigers.

7. a. Let matrix A represent the labour hours at Cuddly Toy Company.

$$\therefore\ A = \begin{bmatrix} 0.5 & 0.8 & 0.4 \\ 0.8 & 1.0 & 0.5 \\ 0.6 & 0.4 & 0.5 \end{bmatrix}$$

b. Let matrix B represent the orders for October and November.



$$\therefore\ B = \begin{bmatrix} 1000 & 1100 \\ 600 & 850 \\ 800 & 725 \end{bmatrix}$$

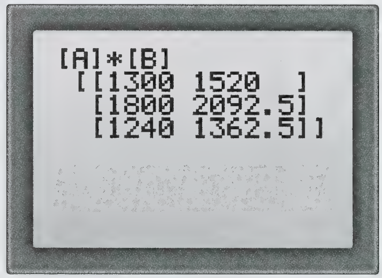
c. The order of matrix multiplication in exercise 7.c. has been specified as $A \times B$. You need to write matrix B with three rows because there must be the same number of rows in matrix B as there are columns in matrix A.

Method 1: Using Pencil and Paper

$$A \times B = \begin{bmatrix} 0.5 & 0.8 & 0.4 \\ 0.8 & 1.0 & 0.5 \\ 0.6 & 0.4 & 0.5 \end{bmatrix} \times \begin{bmatrix} 1000 & 1100 \\ 600 & 850 \\ 800 & 725 \end{bmatrix}$$

$$= \begin{bmatrix} 1300 & 1520 \\ 1800 & 2092.5 \\ 1240 & 1362.5 \end{bmatrix}$$

Method 2: Using a Graphing Calculator



To fill the orders, 4340 labour hours are required for October and 4975 labour hours are required for November.

d. Let matrix C represent the labour rates in each plant.

$$\therefore\ C = \begin{bmatrix} 8.50 & 9.00 & 8.40 \\ 8.00 & 8.50 & 8.20 \\ 8.90 & 10.50 & 10.15 \end{bmatrix}$$

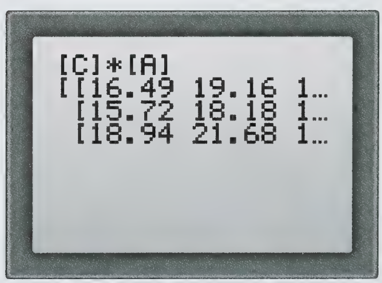
The product of $C \times A$ will show the costs of producing each item in each plant. This is because, in the contex of the problem, you need to multiply the time for each toy by the cost of each task. The matrix operation sums these products as the total cost of producing a particular toy in a particular plant (province). If you were to multiply $A \times C$, you would be multiplying the time for each toy by the cost of one task in each of the plants (provinces). This is incorrect.

Method 1: Using Pencil and Paper

$$C \times A = \begin{bmatrix} 8.50 & 9.00 & 8.40 \\ 8.00 & 8.50 & 8.20 \\ 8.90 & 10.50 & 10.15 \end{bmatrix} \times \begin{bmatrix} 0.5 & 0.8 & 0.4 \\ 0.8 & 1.0 & 0.5 \\ 0.6 & 0.4 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 16.49 & 19.16 & 12.10 \\ 15.72 & 18.18 & 11.55 \\ 18.94 & 21.68 & 13.89 \end{bmatrix}$$

Method 2: Using a Graphing Calculator



The following table shows the labour costs in each plant.

	[illegible]	Kangaroo	[illegible]
Alberta	\$16.49	\$19.16	\$12.10
Saskatchewan	\$15.72	\$18.18	\$11.55
Manitoba	\$18.94	\$21.68	\$13.89

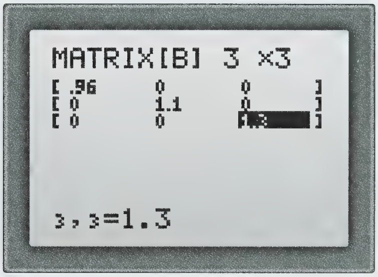
9. **a.** Let matrix A represent the 1998 sales.

$\therefore A = \begin{bmatrix} 200 & 300 & 40 \end{bmatrix}$

b. Let matrix B represent the matrix for the change in sales from 1998 to 1999.

$$\therefore B = \begin{bmatrix} 100\% - 4\% & 0 & 0 \\ 0 & 100\% + 10\% & 0 \\ 0 & 0 & 100\% + 30\% \end{bmatrix}$$

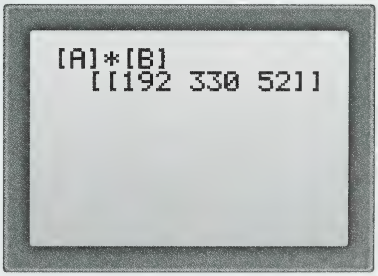
$$= \begin{bmatrix} 0.96 & 0 & 0 \\ 0 & 1.10 & 0 \\ 0 & 0 & 1.30 \end{bmatrix}$$



c. The car sales in 1999 is determined by finding the product of $A \times B$.

$\therefore A \times B = \begin{bmatrix} 192 & 330 & 52 \end{bmatrix}$

In 1999, the sales of economy cars were 192, the sales of mid-size cars were 330, and the sales of luxury cars were 52.



5. Two types of branches required to construct a chair are 1- to 2-inch (2.5- to 5.0-cm) diameter hardwood pieces for the legs and seat frame and $\frac{3}{4}$-inch (1.5-cm) diameter pliable lengths for the seat and back.

6. The branches are fastened together with nails or screws or by cutting "tenon" joints, which are glued together.

7. **Table A**

	[illegible]	[illegible]	[illegible]
3-foot Hardwood	15	15	20
5-foot Hardwood	6	5	0
5-foot Pliable	14	10	4
8-foot Pliable	0	3	0
Total	35	33	24

$$\therefore A = \begin{bmatrix} 15 & 15 & 20 \\ 6 & 5 & 0 \\ 14 & 10 & 4 \\ 0 & 3 & 0 \\ 35 & 33 & 24 \end{bmatrix}$$

8. a. $B = \begin{bmatrix} 4 & 9 & 7 & 2 \\ 2 & 4 & 3 & 5 \\ 3 & 6 & 7 & 3 \end{bmatrix}$

b. **Project Book exercises 1.a. to 1.d. of "Getting Started," p. 23**

1. a. The order of multiplication will be $A \times B$.

   b. The dimensions of the product are five rows by four columns, or 5×4.

   c. Matrix C is a Size-Month matrix.

   d. The entries of matrix C represent the number of branches of each type required for each month.

9. Project Book exercise 2 of "Getting Started," pp. 23 and 24

2. **Collection:** $1\,\text{h} / 30 = 0.033\,\text{h}$ or $60\,\text{min}/ 30 = 2\,\text{min}$
   Construction: $4\,\text{h} / 30 = 0.133\,\text{h}$ or $240\,\text{min}/ 30 = 8\,\text{min}$
   Finishing: $1\,\text{h} / 30 = 0.033\,\text{h}$ or $60\,\text{min}/ 30 = 2\,\text{min}$

10. Project Book exercises 3.a. to 3.c. of "Getting Started," p. 24

3. **a.** To get each row in matrix D, multiply each entry from the fifth row of matrix A by the time required for the particular process.

Table D

	[illegible]	Settee	[illegible]
Collection	$35\times0.033=1.155$	$33\times0.033=1.089$	$24\times0.033=0.792$
Construction	$35\times0.133=4.655$	$33\times0.133=4.389$	$24\times0.133=3.192$
Finishing	$35\times0.033=1.155$	$33\times0.033=1.089$	$24\times0.033=0.792$

b. The dimensions of matrix D are 3×3.

c. Matrix D is an Item-Process matrix.

Table D

	[illegible]	Settee	[illegible]
Collection	1.155	1.089	0.792
Construction	4.655	4.389	3.192
Finishing	1.155	1.089	0.792

11. $E=\begin{bmatrix}7 & 10 & 10\end{bmatrix}$

12. Project Book exercises 4.a. to 4.c. of "Getting Started," p. 24

4. **a.** The order of multiplication will be $E\times D$.
   b. The dimensions of matrix F are 1×3.
   c. The entries in matrix F represent the total cost of the production process for the different items.

13. $G=\begin{bmatrix}6.25 & 9.25 & 3.20\end{bmatrix}$

14. Matrix H can be obtained by adding matrices F and G. But, first, you must determine matrix F.

$$
\begin{aligned}
F &= E \times D \\
&= \begin{bmatrix} 7 & 10 & 10 \end{bmatrix} \times \begin{bmatrix} 1.155 & 1.089 & 0.792 \\ 4.655 & 4.389 & 3.192 \\ 1.155 & 1.089 & 0.792 \end{bmatrix} \\
&= \begin{bmatrix} 66.185 & 62.403 & 45.384 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\therefore\ H &= F + G \\
&= \begin{bmatrix} 66.185 & 62.403 & 45.384 \end{bmatrix} + \begin{bmatrix} 6.25 & 9.25 & 3.20 \end{bmatrix} \\
&= \begin{bmatrix} 72.435 & 71.653 & 48.584 \end{bmatrix}
\end{aligned}
$$

15. Project Book exercises 6, 7, and 8 of "Getting Started," p. 25

6. To obtain matrix J, multiply matrix H by the scalar 1.25.

7. To obtain matrix K, subtract matrix H from matrix J.

8. **a.** The order of the matrix multiplication will be $K \times B$.
b. The dimensions of the product matrix will be 1×4.
c. The information found in the entries of matrix L is total profit for each type of item each month.

16. Textbook exercise "Communicating the Ideas," p. 67

Diagrams will vary. A sample diagram is given.

$$
\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}
$$

Activity 3: Solving Network Problems with Matrices

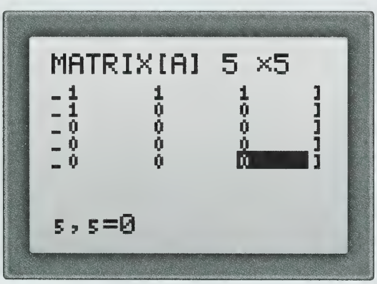
1. Textbook exercises 1 to 7 of "Investigation: Determining the Number of Stopovers," p. 73

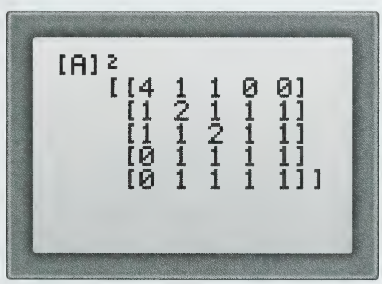
1. Number of Routes Between Cities with Exactly One Stopover

		TO City				
		[illegible]	Inuvik	[illegible]	Rankin Inlet	[illegible]
FROM City	[illegible]	4	1	1	0	0
	Inuvik	1	2	1	1	1
	[illegible]	1	1	2	1	1
	Rankin Inlet	0	1	1	1	1
	[illegible]	0	1	1	1	1

$$\therefore\ B = \begin{bmatrix} 4 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

2.



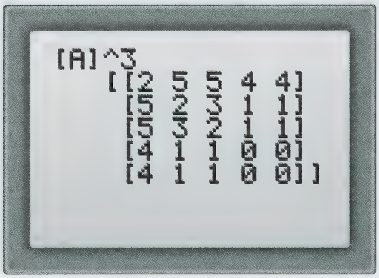


A^2 and B are the same.

3. Let matrix C represent the number of routes between cities with exactly 2 stopovers.

$$\therefore\ C = \begin{bmatrix} 2 & 5 & 5 & 4 & 4 \\ 5 & 2 & 3 & 1 & 1 \\ 5 & 3 & 2 & 1 & 1 \\ 4 & 1 & 1 & 0 & 0 \\ 4 & 1 & 1 & 0 & 0 \end{bmatrix}$$

4.



A^3 and C are the same.

5. According to the pattern, calculate A^5 to obtain the matrix that represents the number of routes between cities with exactly 4 stopovers.

6. $$A + A^2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 2 & 1 & 1 \\ 2 & 2 & 2 & 1 & 1 \\ 2 & 2 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

This matrix, $A + A^2$, represents the number of routes between cities with 1 stopover or less (or no more than 1 stopover).

7. At most, 2 stopovers means 0, 1, or 2 stopovers. To find the matrix that represents the number of routes between cities with at most 2 stopovers, find the sum of matrices A, A^2, and A^3, which represent 0 stopovers, 1 stopover, and 2 stopovers, respectively.

2. Textbook exercises 1, 2.a., 5, 6, and 7 of "Exercises: Checking Your Skills," pp. 76 to 79

1. Create a table. Make sure the "FROM" list and the "TO" list show the points in the same order.

a.

FROM \ TO	A	B	C	D
A	0	1	0	1
B	1	0	0	1
C	1	0	0	0
D	0	1	1	1

$$\therefore \text{Matrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

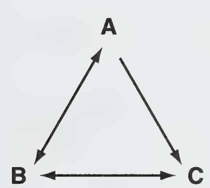
b.

FROM \ TO	W	X	Y
W	0	1	1
X	0	0	1
Y	1	0	0

$$\therefore \text{Matrix} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

2. a. Create a table from the matrix. Then use the table to draw the network.

FROM \ TO	A	B	C
A	0	1	1
B	1	0	1
C	0	1	0



5. **a.** Set up a table to represent the communications system. Let 1 represent direct communication between individuals and 0 represent no direct communication.

		TO						
		CEO	VPS	VPM	VPF	RSR	RMR	RA
FROM	CEO	0	1	1	1	0	0	0
	VPS	0	0	1	0	1	1	0
	VPM	0	1	0	1	1	1	0
	VPF	0	0	1	0	0	0	1
	RSR	0	0	0	0	0	0	0
	RMR	0	0	0	0	0	0	0
	RA	0	0	0	0	0	0	0

CEO: Chief Executive Officer
VPS: Vice-President (Sales)
VPM: Vice-President (Marketing)
VPF: Vice-President (Finance)
RSR: Regional Sales Representative
RMR: Regional Marketing Representative
RA: Regional Accountant

Let A represent the network matrix.

$$\therefore A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

b. The matrix A^2 represents the number of ways an individual can reach another with exactly one intermediary.

Using your graphing calculator,

CEO to RSR

$$A^2 = \begin{bmatrix} 0 & 1 & 2 & 1 & 2 & 2 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

There are 2 ways the CEO can send a message to the regional sales representative with exactly one intermediary.

c. The matrix $A + A^2 + A^3$ represents the number of ways an individual can reach another with at most two intermediaries. Use your graphing calculator to determine this matrix.

CEO to RMR

$$A + A^2 + A^3 = \begin{bmatrix} 0 & 4 & 5 & 4 & 5 & 5 & 2 \\ 0 & 1 & 3 & 1 & 3 & 3 & 1 \\ 0 & 3 & 2 & 3 & 4 & 4 & 1 \\ 0 & 1 & 3 & 1 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

MATRX 1 (1:[A] 7×7) + MATRX
1 (1:[A] 7×7) x^2 + MATRX
1 (1:[A] 7×7) ^ 3 ENTER

There are 5 ways the CEO can send a message to the regional marketing representative with at most two intermediaries.

6. **a.** Set up a table to represent the network. Let 1 represent a direct route from one destination to another and 0 represent no direct route.

		TO						
		L	M	C	W	G	CH	T
FROM	L	0	1	0	0	1	0	0
	M	0	0	1	0	0	1	0
	C	0	0	0	1	0	1	1
	W	0	0	0	0	0	0	1
	G	1	1	0	0	0	0	0
	CH	0	1	1	0	1	0	0
	T	0	0	0	1	0	1	0

L: Library
M: Museum
C: City Hall
W: Waterfront Park
G: Gallery
CH: Court House
T: Tourist Information

Let A be the network matrix.

$$\therefore A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

b. The matrix A^4 represents the number of ways of going from one destination to another with exactly three stops on the way.

$$A^4 = \begin{bmatrix} 2 & 4 & 4 & 3 & 2 & 5 & 3 \\ 1 & 5 & 5 & 4 & 4 & 6 & 4 \\ 1 & 5 & 5 & 4 & 4 & 6 & 4 \\ 1 & 1 & 1 & 2 & 0 & 3 & 1 \\ 1 & 4 & 4 & 3 & 3 & 5 & 3 \\ 3 & 6 & 6 & 5 & 3 & 8 & 5 \\ 0 & 4 & 4 & 2 & 4 & 3 & 3 \end{bmatrix} \begin{array}{l} \\ \\ \\ \\ \leftarrow \text{Gallery to Tourist Information} \\ \\ \\ \end{array}$$

There are 3 ways a tourist could get from the Gallery to the Tourist Information centre and see exactly three sites along the way.

7. Let A be the network matrix corresponding to the given table.

$$\therefore A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

a. The matrix A^2 represents the ways you can travel with exactly one stop.

$$A^2 = \begin{bmatrix} 2 & 1 & 2 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 2 & 1 & 1 & 2 \\ 2 & 1 & 3 & 1 & 2 & 2 & 2 \\ 1 & 2 & 1 & 4 & 2 & 2 & 3 \\ 1 & 1 & 2 & 2 & 3 & 2 & 2 \\ 1 & 1 & 2 & 2 & 2 & 3 & 2 \\ 1 & 2 & 2 & 3 & 2 & 2 & 6 \end{bmatrix} \begin{array}{l} \\ \\ \\ \\ \\ \leftarrow \text{Thunder Bay to Calgary} \\ \\ \end{array}$$

There are 2 ways you can travel from Thunder Bay to Calgary with exactly one stop.

b. The matrix $A + A^2 + A^3$ represents the ways you can travel with at most two stops.

$$A + A^2 + A^3 = \begin{bmatrix} 4 & 7 & 5 & 6 & 4 & 4 & 10 \\ 7 & 7 & 9 & 7 & 6 & 6 & 12 \\ 5 & 9 & 7 & 11 & 7 & 7 & 14 \\ 6 & 7 & 11 & 12 & 12 & 12 & 16 \\ 4 & 6 & 7 & 12 & 9 & 10 & 14 \\ 4 & 6 & 7 & 12 & 10 & 9 & 14 \\ 10 & 12 & 14 & 16 & 14 & 14 & 18 \end{bmatrix}$$

Thunder Bay to Prince George

There are 4 ways you can travel from Thunder Bay to Prince George with at most two stops.

c. First, revise matrix A to show a direct flight between Edmonton and Prince George. These changes are bolded.

$$A = \begin{bmatrix} 0 & 1 & \mathbf{1} & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ \mathbf{1} & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Determine $A + A^2 + A^3$.

$$A + A^2 + A^3 = \begin{bmatrix} 9 & 10 & 12 & 8 & 6 & 6 & 14 \\ 10 & 9 & 12 & 8 & 6 & 6 & 14 \\ 12 & 12 & 12 & 13 & 8 & 8 & 17 \\ 8 & 8 & 13 & 12 & 12 & 12 & 17 \\ 6 & 6 & 8 & 12 & 9 & 10 & 14 \\ 6 & 6 & 8 & 12 & 10 & 9 & 14 \\ 14 & 14 & 17 & 17 & 14 & 14 & 20 \end{bmatrix}$$

If a direct flight from Edmonton to Prince George is added, there would be 6 ways you can travel from Thunder Bay to Prince George with at most two stops.

d. With the Calgary airport temporarily closed, you must revise matrix A (the original matrix, not the revised matrix A from exercise 7.c.) to show that there are no flights departing or arriving in Calgary.

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \mathbf{0} \\ 1 & 0 & 1 & 0 & 0 & 0 & \mathbf{0} \\ 0 & 1 & 0 & 1 & 0 & 0 & \mathbf{0} \\ 0 & 0 & 1 & 0 & 1 & 1 & \mathbf{0} \\ 0 & 0 & 0 & 1 & 0 & 1 & \mathbf{0} \\ 0 & 0 & 0 & 1 & 1 & 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Determine $A+A^2+A^3$.

Vancou

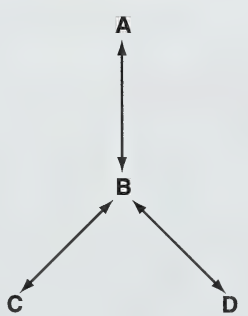
$$A+A^2+A^3=\begin{bmatrix} 1 & 3 & 1 & 1 & 0 & 0 & 0 \\ 3 & 2 & 4 & 1 & 1 & 1 & 0 \\ 1 & 4 & 2 & 5 & 2 & 2 & 0 \\ 1 & 1 & 5 & 5 & 6 & 6 & 0 \\ 0 & 1 & 2 & 6 & 4 & 5 & 0 \\ 0 & 1 & 2 & 6 & 5 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

With the Calgary airport temporarily closed, there is only 1 way to get from Vancouver to Winnipeg with at most two stops.

3. Textbook exercise "Communicating the Ideas," p. 79

Answers may vary. A sample answer is given.

Spy Network



First, set up a table representing the spy network.

		TO			
		A	B	C	D
FROM	A	0	1	0	0
	B	1	0	1	1
	C	0	1	0	0
	D	0	1	0	0

Let N be the network matrix.

$$\therefore\ N = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

The matrix $N + N^2$ represents the number of ways a message can be sent with at most one intermediate spy. Use your graphing calculator to determine this matrix.

$$N + N^2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Create a table showing the number of ways of sending a message with at most one intermediate spy.

		TO			
		A	B	C	D
FROM	A	1	1	1	1
	B	1	3	1	1
	C	1	1	1	1
	D	1	1	1	1

Activity 4: Solving Transition Problems with Matrices

1. Textbook exercises 1 to 7 of "Investigation: Finding *n*-Step Probabilities," pp. 80 and 81

1. $P_1 = P_0 \times T$

$$= \begin{bmatrix} 0.50 & 0.50 \end{bmatrix} \times \begin{bmatrix} 0.65 & 0.35 \\ 0.30 & 0.70 \end{bmatrix}$$

$$= \begin{bmatrix} (0.50 \times 0.65) + (0.50 \times 0.30) & (0.50 \times 0.35) + (0.50 \times 0.70) \end{bmatrix}$$

$$= \begin{bmatrix} 0.475 & 0.525 \end{bmatrix}$$

The elements in P_1 represent the probabilities of Dana and Joe winning the second of a series of squash games.

2. $P_2 = P_1 \times T$

$$= \begin{bmatrix} 0.475 & 0.525 \end{bmatrix} \times \begin{bmatrix} 0.65 & 0.35 \\ 0.30 & 0.70 \end{bmatrix}$$

$$= \begin{bmatrix} 0.466\,25 & 0.533\,75 \end{bmatrix}$$

The elements in P_2 represent the probabilities of Dana and Joe winning the third of a series of squash games.

3. $P_3 = P_2 \times T$

$$= \begin{bmatrix} 0.466\,25 & 0.533\,75 \end{bmatrix} \times \begin{bmatrix} 0.65 & 0.35 \\ 0.30 & 0.70 \end{bmatrix}$$

$$= \begin{bmatrix} 0.463\,187\,5 & 0.536\,812\,5 \end{bmatrix}$$

The elements in P_3 represent the probabilities of Dana and Joe winning the fourth of a series of squash games.

4. $P_4 = P_3 \times T$

$$= \begin{bmatrix} 0.463\,187\,5 & 0.536\,812\,5 \end{bmatrix} \times \begin{bmatrix} 0.65 & 0.35 \\ 0.30 & 0.70 \end{bmatrix}$$

$$= \begin{bmatrix} 0.462\,115\,625 & 0.537\,884\,375 \end{bmatrix}$$

The elements in P_4 represent the probabilities of Dana and Joe winning the fifth of a series of squash games.

5. $$P_0 \times T^2 = \begin{bmatrix} 0.50 & 0.50 \end{bmatrix} \times \begin{bmatrix} 0.65 & 0.35 \\ 0.30 & 0.70 \end{bmatrix}^2$$
$$= \begin{bmatrix} 0.50 & 0.50 \end{bmatrix} \times \begin{bmatrix} 0.5275 & 0.4725 \\ 0.4050 & 0.5950 \end{bmatrix}$$
$$= \begin{bmatrix} 0.466\,25 & 0.533\,75 \end{bmatrix}$$

This result is the same as P_2 in exercise 2.

6. $$P_0 \times T^3 = \begin{bmatrix} 0.50 & 0.50 \end{bmatrix} \times \begin{bmatrix} 0.65 & 0.35 \\ 0.30 & 0.70 \end{bmatrix}^3$$
$$= \begin{bmatrix} 0.50 & 0.50 \end{bmatrix} \times \begin{bmatrix} 0.484\,625 & 0.515\,375 \\ 0.441\,750 & 0.558\,250 \end{bmatrix}$$
$$= \begin{bmatrix} 0.463\,187\,5 & 0.536\,812\,5 \end{bmatrix}$$

This result is the same as P_3 in exercise 3.

$$P_0 \times T^4 = \begin{bmatrix} 0.50 & 0.50 \end{bmatrix} \times \begin{bmatrix} 0.65 & 0.35 \\ 0.30 & 0.70 \end{bmatrix}^4$$
$$= \begin{bmatrix} 0.50 & 0.50 \end{bmatrix} \times \begin{bmatrix} 0.469\,618\,75 & 0.530\,381\,25 \\ 0.454\,612\,50 & 0.545\,387\,50 \end{bmatrix}$$
$$= \begin{bmatrix} 0.462\,115\,625 & 0.537\,884\,375 \end{bmatrix}$$

This result is the same as P_4 in exercise 4.

7. The probabilities after 20 games can be found by calculating the probabilities of each game $\left(P_1, P_2, \ldots, P_{20}\right)$ and then using P_{20}. The probabilities can also be found by calculating $P_0 \times T^{20}$ and using the results for the probabilities.

The first method mentioned is best if you need to know the probabilities for the intermediate numbers of games. The second method mentioned is best if you only need the results for probabilities after 20 games. This second method would be quicker.

2. **Textbook exercises 1, 2, and 3 of "Discussing the Ideas," p. 83**

1. As you project farther into the future, your predictions grow less reliable. The projected population shifts may be accurate today, but they are subject to change depending on a variety of environmental, economic, demographic, and lifestyle factors.

2. Your predictions are only as reliable as your transition matrix. If the elements of the transition matrix are estimates, predictions will be subject to progressively larger errors.

3. The probabilities of an event occurring and an event not occurring always add up to 100% or 1. The rows of a probability matrix and of a transition matrix are the probabilities of complementary events. For example, if the probability that John will win his chess game is $\frac{1}{6}$, the probability that he will lose is $\frac{5}{6}$.

$$\begin{aligned} P(\text{win})+P(\text{loss}) &= \frac{1}{6}+\frac{5}{6} \\ &= 1 \end{aligned}$$

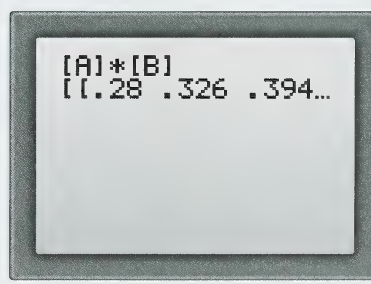
3. a. **Textbook exercises 5, 6, and 7 of "Exercises: Checking Your Skills," p. 84**

5. a. $$P_0 = \begin{array}{c} \begin{array}{ccc} \text{full-size} & \text{mid-size} & \text{compact} \end{array} \\ \begin{bmatrix} 0.30 & 0.20 & 0.50 \end{bmatrix} \end{array}$$

b. $$T = \begin{array}{cl} \begin{array}{ccc} \text{full-size} & \text{mid-size} & \text{compact} \end{array} & \\ \begin{bmatrix} 0.85 & 0.13 & 0.02 \\ 0.05 & 0.91 & 0.04 \\ 0.03 & 0.21 & 0.76 \end{bmatrix} & \begin{array}{l} \leftarrow \text{first-time, full-size buyers} \\ \leftarrow \text{first-time, mid-size buyers} \\ \leftarrow \text{first-time, compact buyers} \end{array} \end{array}$$

c. P_1 is market share for second-time buyers.

$$\begin{aligned} \therefore\ P_1 &= P_0 \times T \\ &= \begin{bmatrix} 0.30 & 0.20 & 0.50 \end{bmatrix} \times \begin{bmatrix} 0.85 & 0.13 & 0.02 \\ 0.05 & 0.91 & 0.04 \\ 0.03 & 0.21 & 0.76 \end{bmatrix} \\ &= \begin{bmatrix} 0.28 & 0.326 & 0.394 \end{bmatrix} \end{aligned}$$



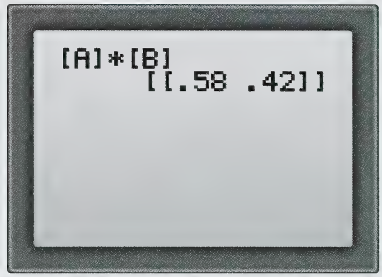
The market share for second-time buyers is
28% full-size, 32.6% mid-size, and 39.4% compact.

6. **a.** $$P_0 = \begin{array}{cc} \text{win} & \text{lose} \\ \end{array}$$
$$P_0 = \begin{bmatrix} 0.60 & 0.40 \end{bmatrix}$$

$$\begin{array}{ccc} & \text{win} & \text{lose} & \\ T = & \left[\begin{array}{cc} 0.80 & 0.20 \\ 0.25 & 0.75 \end{array}\right] & & \begin{array}{l} \leftarrow \text{wins first match} \\ \leftarrow \text{loses first match} \end{array} \end{array}$$

b. The probability matrix for the second match is P_1.

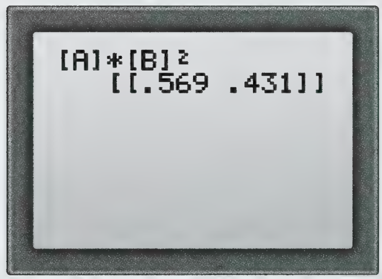
$$P_1 = P_0 \times T$$
$$= \begin{bmatrix} 0.60 & 0.40 \end{bmatrix} \times \begin{bmatrix} 0.80 & 0.20 \\ 0.25 & 0.75 \end{bmatrix}$$
$$= \begin{bmatrix} 0.58 & 0.42 \end{bmatrix}$$



The probability that the team will win its second match is 58%.

c. The probability matrix for the third match is P_2.

$$P_2 = P_0 \times T^2$$
$$= \begin{bmatrix} 0.60 & 0.40 \end{bmatrix} \times \begin{bmatrix} 0.80 & 0.20 \\ 0.25 & 0.75 \end{bmatrix}^2$$
$$= \begin{bmatrix} 0.569 & 0.431 \end{bmatrix}$$



The probability that the team will win its third match is 56.9%.

7. **a.** Determine the percentage of insured drivers who were involved in an accident last year.

$$\text{Percentage in an accident} = \frac{70}{70+249}$$
$$= \frac{70}{319}$$
$$\doteq 0.2194 \text{ or } 21.94\%$$

Therefore, the percentage who were not involved in an accident last year is

$$1 - 0.2194 \doteq 0.7806 \text{ or } 78.06\%$$

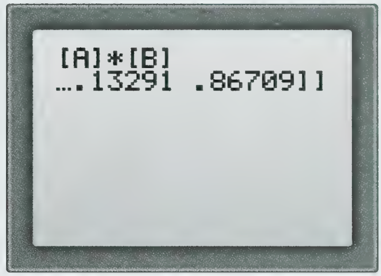
The initial probability matrix, P_0, and the transition matrix, T, are

$$P_0 = \begin{array}{cc} \text{accident} & \text{no accident} \\ \left[0.2194\right. & \left.0.7806\right] \end{array}$$

$$T = \begin{array}{c} \begin{array}{cc} \text{accident} & \text{no accident} \end{array} \\ \begin{bmatrix} 0.25 & 0.75 \\ 0.10 & 0.90 \end{bmatrix} \end{array} \begin{array}{l} \leftarrow \text{accident last year} \\ \leftarrow \text{no accident last year} \end{array}$$

b. The probability matrix for next year's drivers is P_1.

$$\begin{aligned} P_1 &= P_0 \times T \\ &\doteq \begin{bmatrix} 0.2194 & 0.7806 \end{bmatrix} \times \begin{bmatrix} 0.25 & 0.75 \\ 0.10 & 0.90 \end{bmatrix} \\ &\doteq \begin{bmatrix} 0.132\,91 & 0.867\,09 \end{bmatrix} \end{aligned}$$



The percentage of drivers who will have an accident next year is about 0.1329 or 13.29%.

$$\begin{aligned} \therefore \text{ Total number of accidents} &\doteq 0.1329 \times \text{Total number of drivers} \\ &\doteq 0.1329 \times 319 \\ &\doteq 42 \end{aligned}$$

There will be about 42 drivers who will have an accident next year.

b. Textbook exercise 10 of "Exercises: Extending Your Thinking," p. 85

10. a. Determine the percentage of Brand A drinkers and Brand B drinkers.

$$\begin{aligned} \frac{\text{Brand A drinkers}}{\text{Total number of drinkers}} &= \frac{437}{437 + 332} \\ &= \frac{437}{769} \\ &\doteq 0.5683 \text{ or } 56.83\% \end{aligned}$$

Approximately 56.83% of the drinkers surveyed drink Brand A.

Therefore, about $100\% - 56.83\% = 43.17\%$ of the drinkers surveyed drink Brand B.

The initial probability matrix, P_0, and the transition matrix, T, are

$$P_0 \doteq \begin{array}{cc} \text{Brand A} & \text{Brand B} \\ \left[0.5683\right. & \left.0.4317\right] \end{array}$$

$$T = \begin{array}{c} \begin{array}{cc} \text{Brand A} & \text{Brand B} \end{array} \\ \begin{bmatrix} 0.80 & 0.20 \\ 0.40 & 0.60 \end{bmatrix} \end{array} \begin{array}{l} \leftarrow \text{drank Brand A last month} \\ \leftarrow \text{drank Brand B last month} \end{array}$$

b. $P_n = P_0 \times T^n$

Evaluate P_n for successive values of n until the results do not change. This is the steady state.

$P_0 \doteq [0.5683 \quad 0.4317]$
$P_1 \doteq [0.6273 \quad 0.3727]$
$P_2 \doteq [0.6509 \quad 0.3491]$
$P_3 \doteq [0.6604 \quad 0.3396]$

$P_9 \doteq [0.6666 \quad 0.3334]$
$P_{10} \doteq [0.6667 \quad 0.3333]$
$P_{11} \doteq [0.6667 \quad 0.3333]$

On your graphing calculator, you can carry out these calculations quite efficiently if P_0 is stored in matrix A and T is stored in matrix B.

$P_1 = [A][B]$
$P_2 = P_1[B]$ or $[A][B]^2$
$P_3 = P_2[B]$ or $[A][B]^3$

Recall that the calculator stores the answer of the previous calculation; so, the key strokes for each step after P_1 are

× MATRX 2 (2:[B] 2×2) ENTER

The matrix representing the steady state is $[0.6667 \quad 0.3333]$. The final market share is 66.67% for Brand A and 33.33% for Brand B.

4. Textbook exercise "Communicating the Ideas," p. 85

Answers will vary. A sample answer is given.

Transition matrices are used to determine how the probabilities of subsequent events are affected by the occurrence or non-occurrence of the first event. They are useful because they have a number of direct applications in the business world. Two such applications are insurance and marketing.

Module Review

1. Textbook exercise 1 of Part A of "What Should I Be Able to Do?," p. 89

1. a. $A = \begin{array}{c} \begin{array}{cccccccccc} & & & & & QC & NB & & & \end{array} \\ \begin{bmatrix} 0.14 & 0.07 & 0.14 & 0.14 & 0.15 & 0.145 & 0.15 & 0.15 & 0.17 & 0.15 \end{bmatrix} \end{array}$

Answers to 1.b. to 1.d. may vary. Sample answers are given based on the data in the given table.

b. $B = \begin{bmatrix} 2082 \\ 2479 \\ 1873 \\ 1905 \\ 2439 \\ 1986 \\ 1873 \\ 1874 \\ 1870 \\ 2219 \end{bmatrix} \begin{array}{l} \leftarrow \text{British Columbia} \\ \leftarrow \text{Alberta} \\ \leftarrow \text{Saskatchewan} \\ \leftarrow \text{Manitoba} \\ \leftarrow \text{Ontario} \\ \leftarrow \text{Quebec} \\ \leftarrow \text{New Brunswick} \\ \leftarrow \text{Nova Scotia} \\ \leftarrow \text{Prince Edward Island} \\ \leftarrow \text{Newfoundland} \end{array}$

c. Average tax revenue per household = Average yearly clothing expenditure $\times (\text{PST} + \text{GST})$

If you calculate $B \times A$, the elements listed diagonally are the tax amounts needed here.

Province	Average Yearly Clothing Expenditure	Average Tax Revenue Per Household
British Columbia	$2082	$291.48
Alberta	$2479	$173.53
Saskatchewan	$1873	$262.22
Manitoba	$1905	$266.70
Ontario	$2439	$365.85
Quebec	$1986	$287.97
New Brunswick	$1873	$280.95
Nova Scotia	$1874	$281.10
Prince Edward Island	$1870	$317.90
Newfoundland	$2219	$332.85

d. No. The average family in Alberta paid $173.53 in taxes, and the average family in Prince Edward Island paid $317.90 in taxes. The greatest taxes paid per household were by families in Ontario ($365.85), and the lowest taxes paid per household were by families in Alberta.

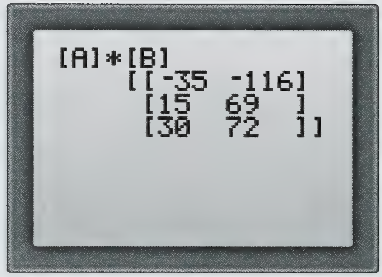
2. Textbook exercises 2.b., 2.d., 5, 8, and 10 of Part B of "What Should I Be Able to Do?," pp. 90 to 92

2. b. Use your graphing calculator.

Enter M as matrix A and N as matrix B.

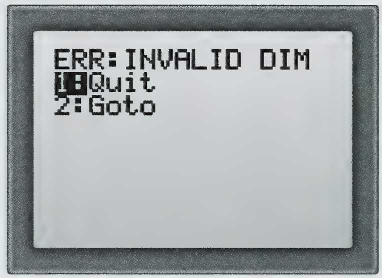
Calculate $M \times N$.

MATRX 1 (1:[A] 3×3) × MATRX
2 (2:[B] 3×2) ENTER



$$\therefore\ M \times N = \begin{bmatrix} -35 & -116 \\ 15 & 69 \\ 30 & 72 \end{bmatrix}$$

d. • 0 5 × MATRX 2 (2:[B] 3×2)
^ 3 ENTER



It is impossible to calculate N^3 because N is not a square matrix.

5. a. Let A be the cost matrix.

$$\therefore\ A = \begin{bmatrix} 8.50 & 11.75 & 15.00 & 24.75 \\ 1.75 & 2.00 & 2.50 & 3.50 \end{bmatrix}$$

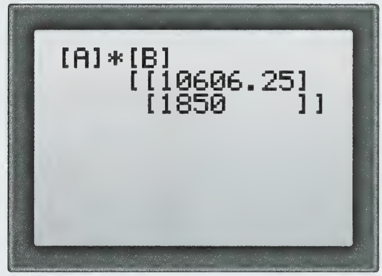
b. Let B be the production matrix.

$$\therefore\ B = \begin{bmatrix} 350 \\ 300 \\ 150 \\ 75 \end{bmatrix}$$

c. The total projected costs are given by the matrix $A \times B$. Use your graphing calculator to determine this product.

$$A \times B = \begin{bmatrix} 8.50 & 11.75 & 15.00 & 24.75 \\ 1.75 & 2.00 & 2.50 & 3.50 \end{bmatrix} \times \begin{bmatrix} 350 \\ 300 \\ 150 \\ 75 \end{bmatrix}$$

$$= \begin{bmatrix} 10\,606.25 \\ 1850.00 \end{bmatrix} \begin{matrix} \longleftarrow \text{materials} \\ \longleftarrow \text{labour} \end{matrix}$$



The entrepreneur's projected costs are \$10 606.25 for materials and \$1850.00 for labour.

8. a. Construct a table to represent this network. Let 1 represent a direct connection and 0 represent no direct connection.

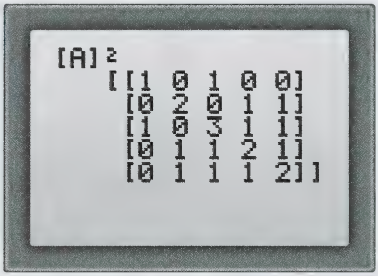
FROM \ TO	A	M	6	7	AS
A	0	1	0	0	0
M	1	0	1	0	0
6	0	1	0	1	1
7	0	0	1	0	1
AS	0	0	1	1	0

A: Argyle Park
M: McMurchy
6: 6th Avenue
7: 7th Avenue
AS: Albert Street

Let A be the matrix representing this table.

$$\therefore A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

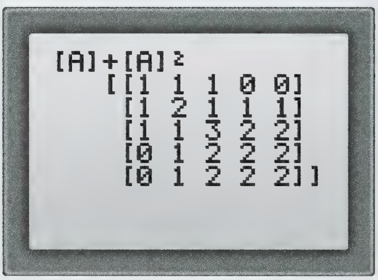
b. Determine A^2 to find the number of ways you can travel from one station to another with exactly 1 relay stop. Use your graphing calculator. Then summarize the results in a table.



With Exactly 1 Stop

		TO				
		[illegible]	M	[illegible]	[illegible]	[illegible]
FROM	[illegible]	1	0	1	0	0
	M	0	2	0	1	1
	[illegible]	1	0	3	1	1
	[illegible]	0	1	1	2	1
	[illegible]	0	1	1	1	2

c. To determine the number of ways you can travel from one station to another with at most 1 relay stop, find $A + A^2$ using your graphing calculator. Then summarize the results in a table.



With at Most 1 Stop

		TO				
		[illegible]	M	[illegible]	[illegible]	[illegible]
FROM	[illegible]	1	1	1	0	0
	M	1	2	1	1	1
	[illegible]	1	1	3	2	2
	[illegible]	0	1	2	2	2
	[illegible]	0	1	2	2	2

d. The matrix $A + A^2 + A^3 + A^4$ represents the number of ways you can travel from one station to another with at most 3 relay stops.

10. **a.** $T = \begin{bmatrix} 0.75 & 0.10 & 0.10 & 0.00 & 0.05 \\ 0.30 & 0.70 & 0.00 & 0.00 & 0.00 \\ 0.25 & 0.00 & 0.65 & 0.10 & 0.00 \\ 0.10 & 0.00 & 0.10 & 0.80 & 0.00 \\ 0.10 & 0.00 & 0.00 & 0.00 & 0.90 \end{bmatrix}$

b. Because a customer is equally likely to borrow from any branch, the probability is $\frac{100\%}{5} = 20\%$ for each branch. Therefore, the initial probability matrix is

$$\begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ P_0 = & \begin{bmatrix} 0.20 & 0.20 & 0.20 & 0.20 & 0.20 \end{bmatrix} \end{array}$$

The probabilities for a customer's second visit are given in the following matrix.

$$P_1 = P_0 \times T$$

$$= \begin{bmatrix} 0.20 & 0.20 & 0.20 & 0.20 & 0.20 \end{bmatrix} \times \begin{bmatrix} 0.75 & 0.10 & 0.10 & 0.00 & 0.05 \\ 0.30 & 0.70 & 0.00 & 0.00 & 0.00 \\ 0.25 & 0.00 & 0.65 & 0.10 & 0.00 \\ 0.10 & 0.00 & 0.10 & 0.80 & 0.00 \\ 0.10 & 0.00 & 0.00 & 0.00 & 0.90 \end{bmatrix}$$

$$\begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ = & \begin{bmatrix} 0.30 & 0.16 & 0.17 & 0.18 & 0.19 \end{bmatrix} \end{array}$$

c. The probabilities for a customer's third visit are given in the following matrix.

$$P_2 = P_0 \times T^2$$

$$= \begin{bmatrix} 0.20 & 0.20 & 0.20 & 0.20 & 0.20 \end{bmatrix} \times \begin{bmatrix} 0.75 & 0.10 & 0.10 & 0.00 & 0.05 \\ 0.30 & 0.70 & 0.00 & 0.00 & 0.00 \\ 0.25 & 0.00 & 0.65 & 0.10 & 0.00 \\ 0.10 & 0.00 & 0.10 & 0.80 & 0.00 \\ 0.10 & 0.00 & 0.00 & 0.00 & 0.90 \end{bmatrix}^2$$

$$\begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ = & \begin{bmatrix} 0.3525 & 0.1420 & 0.1585 & 0.1610 & 0.1860 \end{bmatrix} \end{array}$$

Enrichment

1. Check $x+2y-5z=20$.

LS	RS
$x+2y-5z$	20
$=1+2(2)-5(-3)$	
$=1+4+15$	
$=20$	

LS = RS

Check $2x+y-z=7$.

LS	RS
$2x+y-z$	7
$=2(1)+2-(-3)$	
$=2+2+3$	
$=7$	

LS = RS

Check $x+y+2z=-3$.

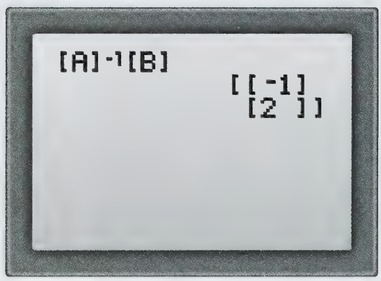
LS	RS
$x+y+2z$	-3
$=1+2+2(-3)$	
$=1+2-6$	
$=-3$	

LS = RS

2. $A=\begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix}$ $\quad B=\begin{bmatrix}5\\ -4\end{bmatrix}$

Now, solve $X=A^{-1}B$.

MATRX 1 x^{-1} MATRX 2 ENTER



$\therefore\ x=-1$ and $y=2$

Check $x+3y=5$.

LS	RS
$x+3y$	5
$=-1+3(2)$	
$=-1+6$	
$=5$	

LS = RS

Check $2x-y=-4$.

LS	RS
$2x-y$	-4
$=2(-1)-2$	
$=-2-2$	
$=-4$	

LS = RS

3. You should rewrite the equation to explicitly show all variables in every equation.

$$w+x+y+z=4$$
$$w+0x+3y+5z=9$$
$$w+0x+0y+z=2$$
$$0w+x-y-z=-1$$

$$\therefore A=\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & 0 & 3 & 5\\ 1 & 0 & 0 & 1\\ 0 & 1 & -1 & -1\end{bmatrix} \qquad B=\begin{bmatrix}4\\ 9\\ 2\\ -1\end{bmatrix}$$

Now, solve for $X=A^{-1}B$.

$\therefore$ $w=1$, $x=1$, $y=1$, and $z=1$

Check $w+x+y+z=4$.

LS	RS
$w+x+y+z$	4
$=1+1+1+1$	
$=4$	

LS = RS

Check $w+3y+5z=9$.

LS	RS
$w+3y+5z$	9
$=1+3(1)+5(1)$	
$=9$	

LS = RS

Check $w+z=2$.

LS	RS
$w+z$	2
$=1+1$	
$=2$	

LS = RS

Check $x-y-z=-1$.

LS	RS
$w-y-z$	-1
$=1-1-1$	
$=-1$	

LS = RS

Module Project: Creating a Long-Distance Telephone Plan

1. **Textbook exercise 11 of Part C of "What Should I Be Able to Do?," p. 93**

   11. Even though she specifies that all values are in dollars, her table and cost matrix are given in cents.

   Also, the TellMe plan charges a flat rate of \$22 for 250 minutes or less of long-distance calls to any place within Canada and the United States on weekday evenings and weekends. The student indicated that, during the time period, her family made 30 minutes of calls within Canada and 40 minutes of calls to the United States. The per-minute charge works out to $\$22/70\,\text{min} = \$0.314/\text{min}$, or $31.4¢/\text{min}$. The per-minute charge she erroneously gives in her table is 10¢/min.

2. **Textbook exercise 12 of Part C of "What Should I Be Able to Do?," p. 94**

   12. She does not specify the hours during which the night-time rates would apply.

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